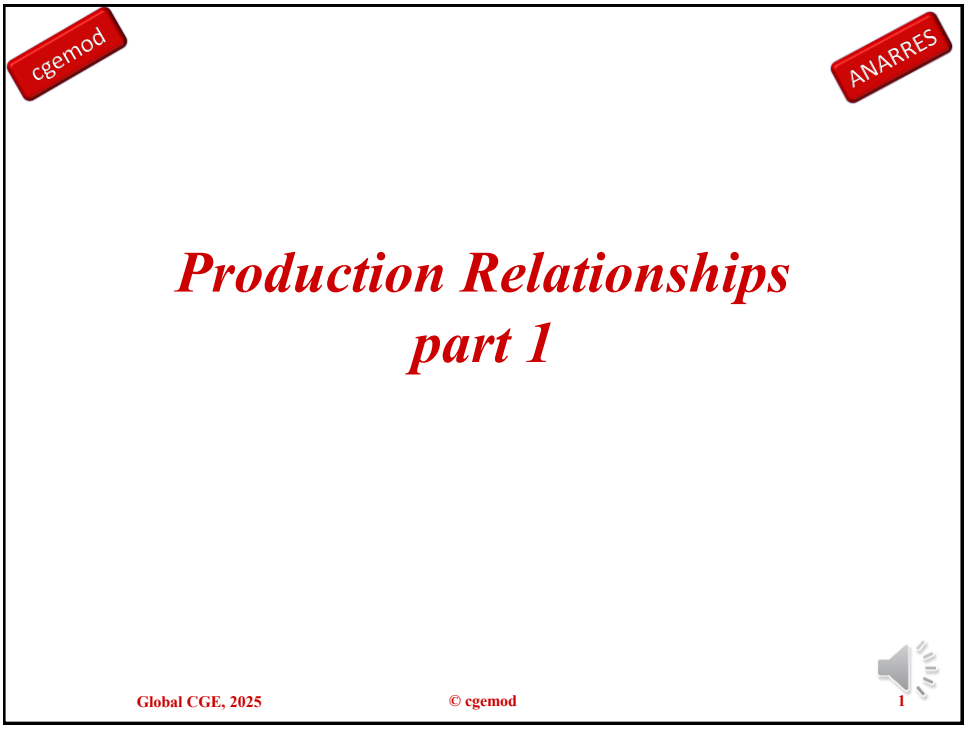
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ANARRES
Production Relationships
part 1
Global CGE, 2025
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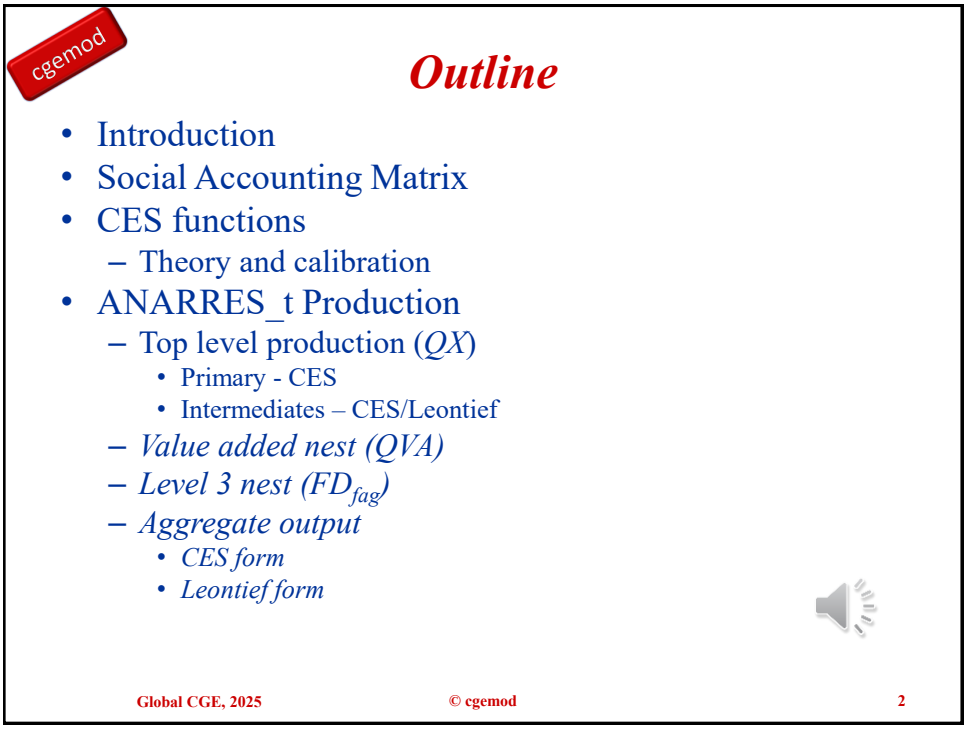
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Outline
• Introduction
• Social Accounting Matrix
• CES functions
– Theory and calibration
• ANARRES_t Production
– Top level production (QX)
• Primary - CES
• Intermediates – CES/Leontief
– Value added nest (QVA)
– Level 3 nest (FDfag)
– Aggregate output
• CES form
• Leontief form
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Social Accounting Matrix

OECD	agr	cnres	cmanu	cserv	aagr	anres	amanu	aserv	fland	fUskil	fskil	fcap	tfland	tfUskil	tfskil	tfcap	HOUS	IMPTAX	EXPTAX	SALTAX	PRODTAX	DIRTAX	Govt	I_S	row	TOTAL
cagr	0	0	0	0	102	0	598	82	0	0	0	0	0	0	0	0	224	0	0	0	0	0	0	9	163	1,179
cnres	0	0	0	0	2	26	1,270	28	0	0	0	0	0	0	0	0	20	0	0	0	0	0	0	9	252	1,608
cmanu	0	0	0	0	222	101	10,733	4,835	0	0	0	0	0	0	0	0	6,871	0	0	0	0	0	51	6,498	6,071	35,382
cserv	0	0	0	0	163	113	4,785	11,034	0	0	0	0	0	0	0	0	16,146	0	0	0	0	0	6,827	1,274	1,963	42,307
aagr	984	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	984
anres	0	726	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	726
amanu	0	0	27,386	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	27,386
aserv	0	0	0	40,137	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40,137
fland	0	0	0	0	120	136	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	256
fUskil	0	0	0	0	207	48	3,331	6,432	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10,018
fskil	0	0	0	0	14	24	1,570	5,760	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7,368
fcap	0	0	0	0	171	218	2,992	8,264	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11,645
tfland	0	0	0	0	-27	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-23
tfUskil	0	0	0	0	30	9	812	1,395	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2,245
tfskil	0	0	0	0	2	5	374	1,291	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,672
tfcap	0	0	0	0	-21	7	78	224	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	287
HOUS	0	0	0	0	0	0	0	0	256	10,018	7,368	7,609	0	0	0	0	0	0	0	0	0	0	0	0	0	25,251
IMPTAX	5	0	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	86
EXPTAX	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
SALTAX	1	11	1,510	508	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2,030
PRODTAX	0	0	0	0	-1	35	842	791	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,668
DIRTAX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4,818	0	0	0	0	0	0	0	0	4,818
Govt	0	0	0	0	0	0	0	0	0	0	0	0	-23	2,245	1,672	287	0	86	1	2,030	1,668	4,818	0	0	0	12,785
I_S	0	0	0	0	0	0	0	0	0	0	0	4,036	0	0	0	0	-2,828	0	0	0	0	0	5,906	0	677	7,791
row	189	871	6,404	1,662	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9,126
TOTAL	1,179	1,608	35,382	42,307	984	726	27,386	40,137	256	10,018	7,368	11,645	-23	2,245	1,672	287	25,251	86	1	2,030	1,668	4,818	12,785	7,791	9,126	

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CES Functions

Theory and calibration

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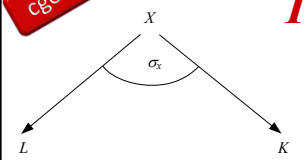
Two Argument CES



$$X = \alpha.\left[\delta * L^{-\rho} + (1-\delta) * K^{-\rho}\right]^{-1/\rho}$$

$$\frac{L}{K} = \left[\frac{\delta}{(1-\delta)}.\frac{PK}{PL}\right]^{\frac{1}{(1+\rho)}}$$

Is the 1st Order condition enough?

Yes, given
1. **Linear homogenous function – Euler**
2. **Chosen functional form**

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Two Argument CES - Calibration

Calibrate the parameters using
1. **base transactions data**
2. **factor quantity data**
3. **output quantities**
4. **exogenously defined elasticities**

```
  rho = (1/ELAST) - 1 ;

delta = (PL0*L0**(rho+1))
              /(PL0*L0**(rho+1) + PK0*K0**(rho+1)) ;

   ac = X0/(delta*L0**(-rho)+(1-delta)*K0**(-rho))
              **(-1/rho) ;
```


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ANARRES_t Production

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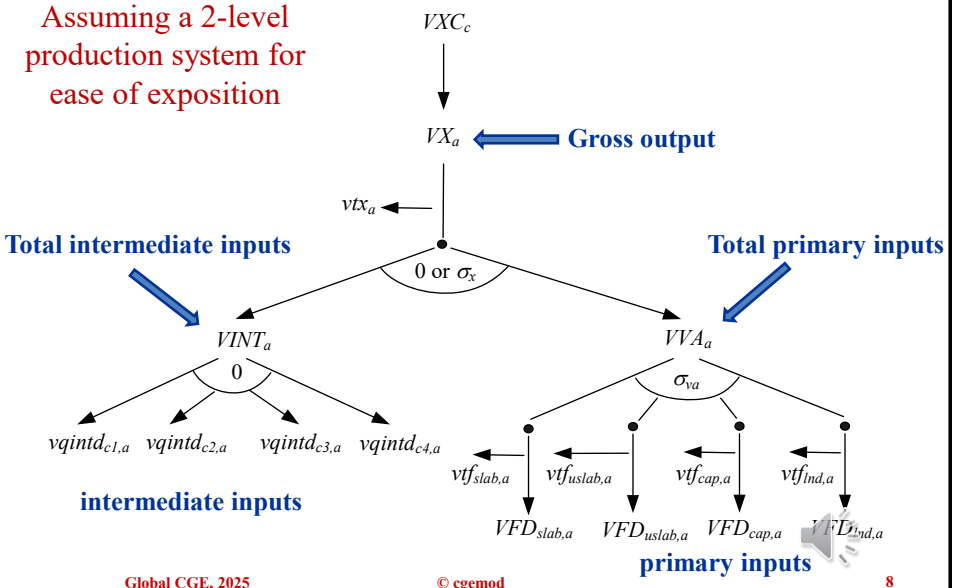
Production – Value System

Assuming a 2-level production system for ease of exposition



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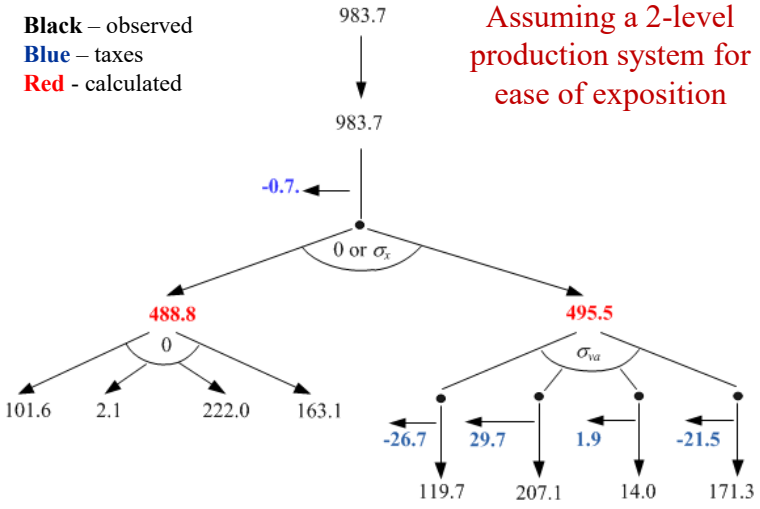
Production - Values

Black – observed
Blue – taxes
Red - calculated

Assuming a 2-level production system for ease of exposition



OECD	aagr
cagr	102
cnres	2
cmanu	222
cserv	163
fland	120
fUskil	207
fskil	14
fcap	171
tfland	-27
tfUskil	30
tfskil	2
tfcap	-21
PRODTAX	-1
TOTAL	984

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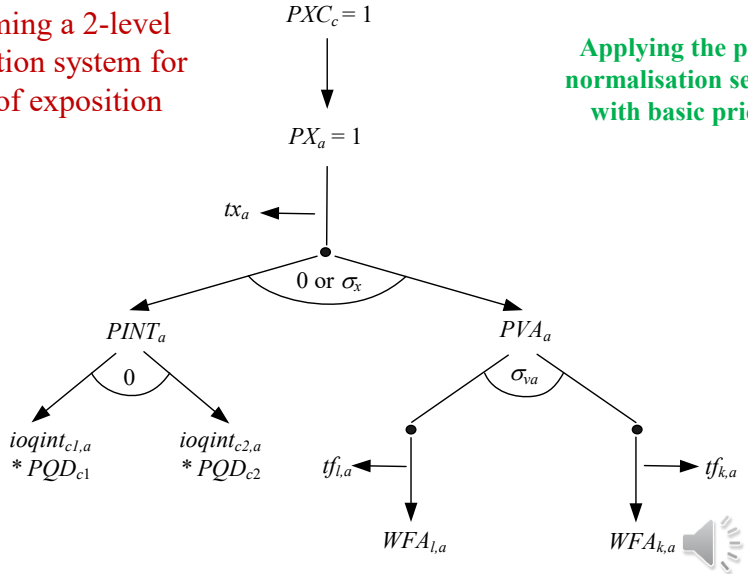
Production - Prices

Assuming a 2-level production system for ease of exposition

Applying the price normalisation setting with basic prices



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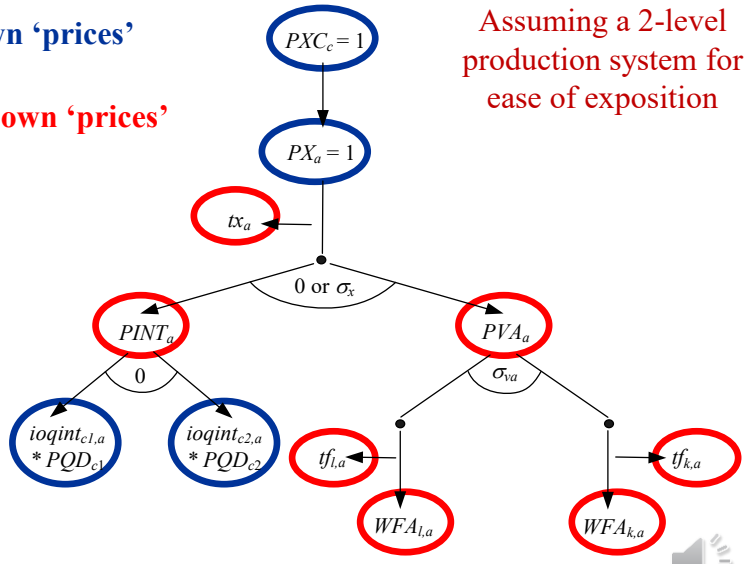
Production - Prices

Known 'prices'

Unknown 'prices'

Assuming a 2-level production system for ease of exposition



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Production - Prices

We do know

$$PXC_c = 1$$
$$PX_a = 1$$
$$PQD_c = PQS_c * (1 + ts_c)$$
$$= 1 + ts_c$$

We do NOT know $WFA_{f,a}, tx_a, tf_{f,a}$

We can NEVER know $PINT_a, PVA_a$

OECD	agr
cagr	102
cnres	2
cmanu	222
cserv	163
fland	120
fUskil	207
fskil	14
fcap	171
tfland	-27
tfUskil	30
tfskil	2
tfcap	-21
PRODTAX	-1
TOTAL	984

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Production - Prices

ASSUME $WFA_{f,a} = 1 \qquad \forall f, a$

Then $FD_{f,a} = VFD_{f,a} \qquad \forall f, a$

These are the prices for the rows of the factor accounts – although so far we have not defined the behaviour that generates them.

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Top Level (Output) Nest

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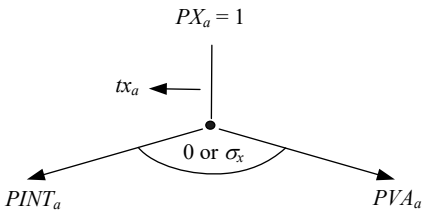
Top Level – Prices



OECD	anagr
cagr	102
cnres	2
cmanu	222
cserv	163
fland	120
fUskil	207
fskil	14
fcap	171
tfland	-27
tfUskil	30
tfskil	2
tfcap	-21
PRODTAX	-1
TOTAL	984

All the components for aggregate value added and aggregate intermediate inputs exist

BUT they must be consistent with Activity Prices and output taxes

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Top Level – CES Version

```
PVADEF(a,r)$rgn(r)..
   PX(a,r)*(1-TX(a,r))*QX(a,r)
          =E= (PVA(a,r)*QVA(a,r)) +
               PINT(a,r)*QINT(a,r)) ;

 PINTDEF(a,r)$rgn(r)..
   PINT(a,r) =E= SUM(c,ioqint(c,a,r) * PQD(c,r)) ;
```


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Top Level – CES Version

```
ADXEQ(a,r)..
  ADX(a,r) =E= ((adxb(a,r) + dabadx(a,r)) *ADXADJ(r))
                 +(DADX(r) * adx01(a,r)) ;

QXPRODFN(a,r)$(aqx(a,r) AND rgn(r))..
  QX(a,r) =E= ADX(a,r)
              *(deltax(a,r)*QVA(a,r)**(-rhox(a,r))
               + (1-deltax(a,r))*QINT(a,r)**(-rhox(a,r)))
                        **(-1/rhox(a,r)) ;

QXFOC(a,r)$(aqx(a,r) AND rgn(r))..
  QVA(a,r) =E= QINT(a,r)
               *((PINT(a,r)/PVA(a,r))
                 *(deltax(a,r)/(1-deltax(a,r))))
                        **(1/(1+rhox(a,r))) ;
```

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Top Level – Calibration

```
rhox(a,r) = (1/mod_elastx(a,r)) - 1 ;

deltax(a,r)$(QINT0(a,r))
     = (PVA0(a,r)*QVA0(a,r)**(1+rhox(a,r)) )
       /(PVA0(a,r)*QVA0(a,r)**(1+rhox(a,r))
          + PINT0(a,r)*QINT0(a,r)**(1+rhox(a,r))) ;

adx0(a,r)$deltax(a,r)
          = QX0(a,r)/(deltax(a,r)*QVA0(a,r)
                        **(-rhox(a,r))
            +(1-deltax(a,r))*QINT0(a,r)**(rhox(a,r)))
                        **(-1/rhox(a,r)) ;
```

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Top Level – Leontief Version

```
QINTDEF(a,r)$(aqxn(a,r) AND rgn(r))..
  QINT(a,r) =E= ioqintqx(a,r) * QX(a,r) ;

QVADEF(a,r)$(aqxn(a,r) AND rgn(r))..
  QVA(a,r) =E= ioqvaqx(a,r) * QX(a,r) ;


ioqintqx(a,r)$QX0(a,r)  =
           QINT0(a,r)/QX0(a,r) ;

ioqvaqx(a,r)$QX0(a,r)  =
           QVA0(a,r)/QX0(a,r) ;
```


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Intermediate Input Nest

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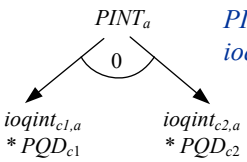
Intermediate Inputs – Prices



PINT is the unit price
$ioqint_{c,a}$ the quantities of input per unit of *QINT*

Omitting the region subscript for clarity

$$QINT_a = \sum_c QINTD_{c,a} = \sum_c \frac{SAM_{c,a}}{PQD_c}$$

$$ioqint_{c,a} = \frac{QINTD_{c,a}}{QINT_a}$$

$$PINT_a = \sum_c ioqint_{c,a} * PQD_c = \sum_c \left(SAM_{c,a} \Big/ PQD_c \Big/ QINT_a \right) * PQD_c$$

OECD	agr
cagr	102
cnres	2
cmanu	222
cserv	163
fland	120
fUskil	207
fskil	14
fcap	171
tfland	-27
tfUskil	30
tfskil	2
tfcap	-21
PRODTAX	-1
TOTAL	984

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Intermediate Inputs – Calibration

```
QINT0(a,r)$SUM(c,SAMG(c,a,r))
                = SUM(c,SAMG(c,a,r)/PQD0(c,r)) ;

PINT0(a,r)$(QINT0(a,r))
              = SUM(c,(SAMG(c,a,r)/PQD0(c,r)
                /QINT0(a,r))* PQD0(c,r)) ;


ioqint(c,a,r)$(QINT0(a,r) AND PQD0(c,r))
              = SAMG(c,a,r)/PQD0(c,r)/QINT0(a,r) ;
```


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Intermediate Inputs – Prices Table

	Transactions				Prices				Quantities			
	aagr	anres	amanu	aserv	aagr	anres	amanu	aserv	aagr	anres	amanu	aserv
						PQD						
cagr	101.6	0.2	598.2	82.4	1.012	1.012	1.012	1.012	100.4	0.2	591.0	81.4
cnres	2.1	26.5	1,270.2	27.9	1.036	1.036	1.036	1.036	2.0	25.6	1,226.6	26.9
cmanu	222.0	100.9	10,732.8	4,835.3	1.080	1.080	1.080	1.080	205.6	93.4	9,938.2	4,477.3
cserv	163.1	113.3	4,785.4	11,034.5	***WFT***	***DIST***	1.008	1.008	159.7	110.9	4,656.9	10,726.9
										QINT		
									466.7	229.4	16411.8	15321.6
	Derived transactions									***ioqint***		
cagr	101.6	0.2	598.2	82.4					0.215	0.001	0.036	0.005
cnres	2.1	26.5	1270.2	27.9					0.004	0.111	0.075	0.002
cmanu	222.0	100.9	10732.8	4835.3					0.441	0.407	0.606	0.292
cserv	163.1	113.3	4785.4	11034.5					0.340	0.480	0.284	0.701
	Derived PINT * QINT					***PINT***						
	198.6	30.1	3,676.1	3,851.6	1.047	1.047	1.057	1.042				
					tx							
prodtax	-3.36	0.60	1.07	170.41	-0.009	0.009	0.000	0.018				
					PX				***QX***			
total	380.22	69.11	5375.10	9458.14	1.000	1.000	1.000	1.000	380.22	69.11	5375.10	9458.14

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Value Added Nest

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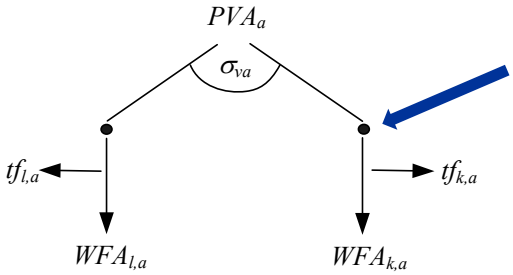
Value Added Nest



Assume this is *PFD(f,a)*

BUT the prices/costs faced by activities when choosing optimal input combinations are inclusive of factor use taxes/subsidies
NOT *WFA(f,a)*

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Value Added - Prices

$$COST_{f,a} = SAMG_{f,a} + SAMG_{tff,a}$$

$$PFD_{f,a} * FD_{f,a} = \left(WFA_{f,a} * FD_{f,a}\right) + \left(tf_{f,a} * WFA_{f,a} * FD_{f,a}\right)$$

$$PFD_{f,a} * FD_{f,a} = \left(WFA_{f,a} * FD_{f,a}\right) + \left(\frac{SAMG_{tff,a}}{SAMG_{f,a}} * WFA_{f,a} * FD_{f,a}\right)$$

$$PFD_{f,a} = \left(WFA_{f,a}\right) + \left(tf_{f,a} * WFA_{f,a}\right) = WFA_{f,a} * \left(1 + tf_{f,a}\right)$$

$$COST_{fland,aagr} = SAMG_{fland,aagr} + SAMG_{tfland,aagr}$$

$$PFD_{fland,aagr} * 119.7 = (1 * 119.7) + \left(\frac{-26.7}{119.7} * 1 * 119.7\right)$$

$$PFD_{f,a} = 1 * \left(1 + (-0.2233891008)\right) = 0.7766108992$$

OECD	aagr
cagr	102
cnres	2
cmanu	222
cserv	163
fland	120
fUskil	207
fskil	14
fcap	171
tfland	-27
tfUskil	30
tfskil	2
tfcap	-21
PRODTAX	-1
TOTAL	984

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Value Added - Prices

Let $QVA0_a = \sum_f SAMG_{f,a}$

$$PVA0_a = \frac{\sum_f SAMG_{f,a} + \sum_{tff} SAMG_{tff,a}}{QVA0_a}$$

$$= \frac{\sum_f PFD_{f,a} * FD_{f,a}}{QVA0_a} = \frac{\sum_f \left(WFA_{f,a} * FD_{f,a}\right) + \left(tf_{f,a} * WFA_{f,a} * FD_{f,a}\right)}{QVA0_a}$$

OECD	aagr
cagr	102
cnres	2
cmanu	222
cserv	163
fland	120
fUskil	207
fskil	14
fcap	171
tfland	-27
tfUskil	30
tfskil	2
tfcap	-21
PRODTAX	-1
TOTAL	984

Hence the prices and quantities for value added are consistent with the transactions values **AND *PVA* derives from the column accounting identity**

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Value Added – Prices Table

	Transactions				Prices				Quantities			
	aagr	anres	amanu	aserv	aagr	anres	amanu	aserv	aagr	anres	amanu	aserv
					PQD							
cagr	101.6	0.2	598.2	82.4	1.012	1.012	1.012	1.012	100.4	0.2	591.0	81.4
cnres	2.1	26.5	1,270.2	27.9	1.036	1.036	1.036	1.036	2.0	25.6	1,226.6	26.9
cmanu	222.0	100.9	10,732.8	4,835.3	1.080	1.080	1.080	1.080	205.6	93.4	9,938.2	4,477.3
cserv	163.1	113.3	4,785.4	11,034.5	1.028	1.028	1.028	1.028	158.7	110.2	4,656.0	10,735.9
					WFA							
fland	119.7	136.2	0.0	0.0	1.000	1.000	1.000	1.000	119.7	136.2	0.0	0.0
fUskil	207.1	47.8	3,331.2	6,431.9	1.000	1.000	1.000	1.000	207.1	47.8	3,331.2	6,431.9
fskil	14.0	23.7	1,570.2	5,759.8	1.000	1.000	1.000	1.000	14.0	23.7	1,570.2	5,759.8
fcap	171.3	218.3	2,991.7	8,264.2	1.000	1.000	1.000	1.000	171.3	218.3	2,991.7	8,264.2
					tf							
tfland	-26.7	4.1	0.0	0.0	-0.223	0.030	0.000	0.000				
tfUskil	29.7	9.0	811.5	1394.5	0.144	0.189	0.244	0.217				
tfskil	1.9	4.7	374.4	1291.2	0.138	0.200	0.238	0.224				
tfcap	-21.5	6.5	78.4	224.1	-0.126	0.030	0.026	0.027				
					PFD							
fland					0.777	1.030	1.000	1.000				
fUskil					1.144	1.189	1.244	1.217				
fskil					1.138	1.200	1.238	1.224				
fcap					0.874	1.030	1.026	1.027				

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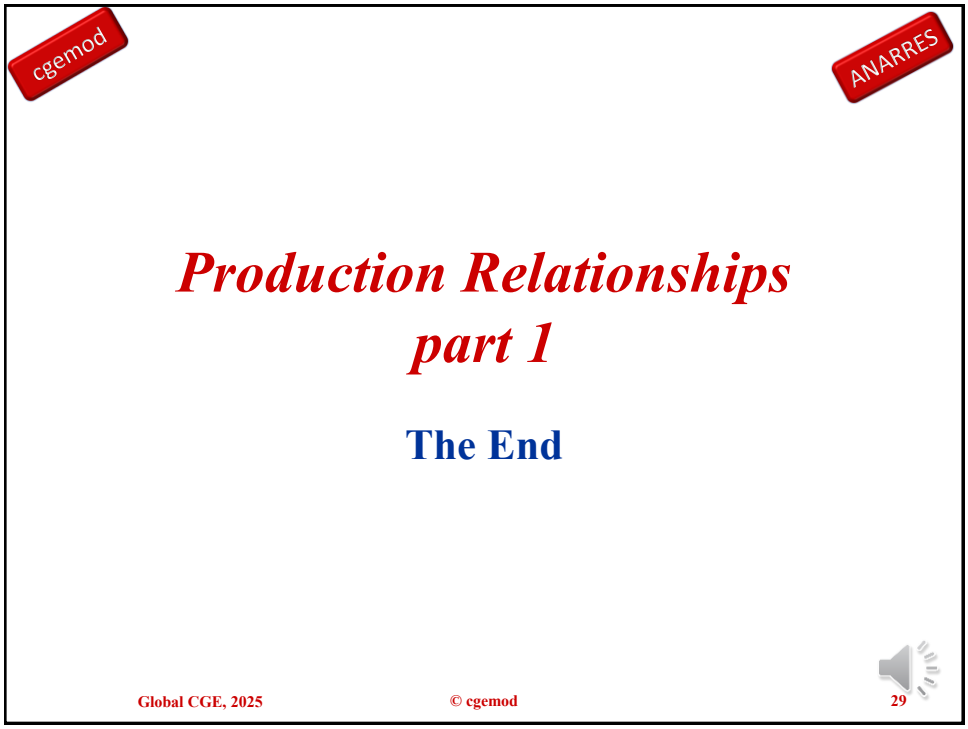
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