Prices, Social Accounts and Economic Models[1]

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Abstract

The relationships between Social Accounting Matrices (SAM) and whole economy models are critical to the design and use of such models. The discussion in this paper explores the role of a price system in national and social accounts and demonstrates how the resultant price definitions require that all whole economy models should obey the 'law of one price'. The relevance of the 'law' to the characteristics of certain features of CGE models is explored and it is demonstrated that while the use of CES functions is consistent with the 'law', the use of CET functions is not consistent with the 'law' although their use may be justified. Three key conclusions are drawn. First, an appropriate representation of the structural relationships in the economy requires and understanding of the price system in a SAM. Second, the construction and/or augmentation of a SAM require an understanding of the price system. And third, any misunderstanding of the price system may lead to incorrect results and flawed policy interpretations.

Keyword: Social Accounting Matrices; CGE models; Prices

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1. Introduction

Underlying national accounts data is a price system that governs how transactions are valued and how the relationships between costs and prices are recorded in the data. [2] Since these data form the empirical basis of whole economy models it is important that the accounting relationships captured in the data are consistent with the economic relationships represented in the models if inconsistencies between the efforts of national account statisticians and economic modellers are to be avoided. Not only are national accounts given their richest specification in Social Accounting Matrices (SAM), it has been long accepted that the databases for all empirical whole economy models are accounting systems (Stone, 1962) and that all such databases can be represented as SAMs (Pyatt, 1987)[3]. Among the class of price driven models, computable general equilibrium (CGE) models are those where the prices are most explicit, and many CGE models record their data explicitly in the form of SAMs, e.g., Dervis *et al*., (1982), Lofgren *et al*., (2001), McDonald and Thierfelder (2004 and 2009).

Despite the fact that SAMs are widely used as the databases for CGE models the relationships between the two appear to be poorly understood; in particular the implications and meaning of the 'law' of one price (LOOP), e.g., the SAMs for CGE models typically record exports as a column of the same commodity accounts as demand, which some users have suggested that this demonstrates that LOOP does not hold. If the price system used to compile the SAM database and the behavioural relationships in the price system of a CGE model are inconsistent there are three clear, and arguably, worrying implications. First, a deficiency in understanding the accounting relationships in a SAM means that when compiling a SAM it is likely, with a high degree of probability, that the structural relationships in the economy may be misrepresented. If such a SAM is then used to calibrate a structural, e.g., CGE, model the resultant analyses may be flawed. Second, any misunderstanding of the behavioural relationships in a CGE model is likely to lead to incorrect results and flawed interpretations of the policy implications of such models. And third, any violations of LOOP within CGE models, even if intentional, are potentially a source of confusion.

The discussion in this paper examines the role of a price system in social accounts and its relationship to the behavioural relationships that determine prices in CGE models. The objectives in the paper are three-fold. First, to demonstrate, in section 3, how the price definitions in CGE models are derived directly from the accounting identities in SAMs and

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2 The price system underlying the UN System of National Accounts (SNA) (ISWGNA, 1993) is the most widely used but arguments have been made for different systems, e.g., Pyatt (1991, 1994a and 1994b).

3 "It is perhaps of interest to realise that the framework of any model concerned with the economy as a whole is always an accounting system. This is true whether we work with highly aggregated models such as that underlying Keynes' General Theory, the input-output model of Leontief or the still more complicated variant with which this series [A Programme for Growth] is concerned." (p v). Pyatt (1987) elaborated on this by demonstrating that "Since every economic model has its corresponding accounting framework, and since every such framework can be set out as a SAM, it follows that every economic model has a corresponding SAM." (p 330)

hence the importance of the relative magnitudes of entries in the columns of a SAM. This section provides the economic theoretic reason why LOOP should hold in CGE models. Second, to explore, in section 4, several common features of CGE models to evaluate the extent to which LOOP is adhered to in CGE models. This section demonstrates that while the application of constant elasticity of substitution (CES) functions is consistent with LOOP, the application of constant elasticity of transformation (CET) functions typically breaches LOOP. And third, to illustrate, in section 5, how an understanding of the price system in a SAM is important when adding additional behavioural relationships to CGE models. This is done using the example of home production for home consumption (HPHC). Throughout the analyses are limited to cases where transactions take place within the SNA's production boundary. The final section, 6, offers some concluding comments, with a call for economists to reengage with national account statisticians so as to ensure the maintenance of an economically consistent price system within national accounts data. The paper starts, section 2, with a description of a SAM; this is based on the SAM that is used for most of the analyses.

2. Social Accounting Matrices

The development of national accounts and SAMs were both largely inspired by Sir Richard Stone.[4] In essence a SAM is a combination of the information contained in aggregate national accounts data, the input-output schema devised by Leontief and disaggregated institutional (social) data, in such a manner as to ensure the full circular flow of an economy is captured. The emphasis on the social dimensions of economic systems is a distinctive feature of SAMs, and is a partial explanation of why they have proved so popular to the study of developing economies. The description of a SAM here is only designed to ease understanding of the subsequent arguments; comprehensive descriptions of SAM are provided by Pyatt (1991, 1994a, 1994b).

2.1 What is a SAM?[5]

A SAM is a system of single entry book keeping presented in the form of a square matrix wherein each account is represented by both a row and a column. The entries in a SAM are transaction values, i.e., prices multiplied by quantities, with the row entries representing incomes to the respective accounts and the columns representing expenditures by the respective accounts. Hence the entry in the i^{th} row and j^{th} column is simultaneously the expenditure by the j^{th} account on the 'product' of the i^{th} account AND the income to the i^{th} account from sales of its product to the j^{th} account. A SAM must 'complete', in the sense that it covers all transactions

4 Stone's contribution to economic and national accounts has been extensively reviewed. See Pesaran and Harcourt (1991) for a review and comprehensive list of publications.

5 The title of this section is taken from King (1985), which still provides one of the best introductions to SAMs.

in an economy, and 'consistent', in the sense that each expenditure by an agent is an identical income for another agent. Hence the total incomes and the total expenditures for every account must equate, i.e.,

$$\sum_i p_{ij}.q_{ij} = \sum_i T_{ij} = \sum_j T_{ij} = \sum_j p_{ij}.q_{ij} \quad \forall i = j \tag{1}$$

where p_{ij} and q_{ij} are the price and quantity of account j used by account i and T_{ij} the transaction (value) between account j and i.

A SAM is flexible arrangement for the collection of economic and social data, and one such arrangement is presented in Table 1.[6] Typically a SAM consists of 6 broad categories of accounts: commodities, activities, factors, institutions (households, incorporated business enterprises, government), capital (for investment and savings), and trade.

The row accounts for commodities identify the distribution of commodities between intermediate and final demand where final demands are disaggregated across different institutions and the capital account. By definition the total demands for commodities are equal to the total supplies of commodities, i.e., the row and column totals equate. The total supplies of commodities include domestic production of commodities by activities, imports and commodity taxes of various types. The commodity accounts therefore trace out the sources and destinations of commodities in the economic system. Two points deserve highlighting; exports, and export taxes, are treated as part of the commodity account, with export taxes included as expenditures by the commodity account, and the SUPPLY matrix allows for activities that produce multiple commodities.[7]

The column entries for activity accounts record the 'use' of inputs by productive activities: intermediate inputs, both domestic and imported, and value added, which are broken down to include payments to different factors and expenditure taxes (subsidies) paid by (to) activities. The column sums for the production accounts record the total inputs to productive activities. In this example, as is common, the row entries only record the output of commodities by activities.

The row entries for factor accounts are incomes to factor accounts for all productive services, including government and domestic services. The sum of these, plus any factor incomes from abroad, are GNP at factor cost. The expenditures by the factor accounts are recorded in the columns. The factor incomes are distributed between domestic institutions as

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6 The representation used here differs from the standard format presented in the SNA (ISWGNA, 1993) by collapsing the institutional accounts. This is a simplification that does not affect the subsequent argument but ensures that the representation is consistent with the format typically used in SAMs to calibrate CGE models and thereby eases exposition. It is trivial to demonstrate that all the subsequent arguments in this paper would hold for a SAM with a SNA (1993) format.

7 Some representations of the SUPPLY matrix of SAMs used to calibrate CGE models include only domestic production supplied to the domestic market; these only have entries on the principal diagonal and record exports in the activity accounts (see Dervis *et al*., 1982); this alternative is a reduced form of the SAM represented in Table 1.

labour income and distributed and non-distributed profits, as tax payments etc., and payments to overseas owned factors.

Table 1 **Structure of a SAM** (about here)

The institutional accounts include private households, enterprises, other institutional entities, e.g., non government organisations, and government. It is common to use representative household groups (RHG) that cluster households according to defined characteristics; for the purposes of these analyses it will be assumed, for generality, that there are multiple RHGs, but this is not the only method of classification or modelling. Incomes to institutions are recorded as row entries and expenditures as column entries. Note how the government is a recipient of multiple forms of income: tax revenues, e.g., tariffs on imports, direct taxes, profit taxes etc., distributed profits and transfers from abroad, e.g., aid.

The Capital account refers to investment and its funding. Commodities in the capital account column record investments whereas the funding of investment is recorded as savings by institutions and capital account transfers from abroad. Finally the rest of the world account records the trade accounts. These include the current and capital accounts, and visible and invisible trade.

2.2 SNA Price Definitions and a SAM

It is important to understand how prices are defined in the SNA since the differences between the prices are critical to the construction of SAMs and the definitions of prices used in economic models calibrated with SAMs. The SNA defines three prices - basic, producer and purchaser – of which two – basic and purchaser – are particularly relevant in the current context.

In essence *basic* prices[8] are the prices received by producers and therefore are the amounts available to cover the unit costs of supplying commodities to an economy from both domestic and foreign (imports) sources; as such they can be conceived of a factory/port gate prices. Transactions in the supply matrix and imported commodities, valued *cif* and inclusive of trade taxes, are valued in terms of basic prices. Consequently *basic* prices are the prices that convey signals to domestic and foreign suppliers and are the output prices that enter into the decision processes of suppliers.

However purchasers must pay prices that allow for any domestic taxes on commodities and for any costs associated with transferring commodities from producers to purchasers. Hence

8 Defined in the SNA as "the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any tax payable, and plus any subsidy receivable, on that unit as a consequence of its production or sale. It excludes any transport charges invoiced separately by the producer" (ISWGNA, 1993, paragraph 205).

purchaser prices[9] are *basic* prices plus commodity taxes and trade and transport margins paid by the purchaser and therefore are the prices that enter into the expenditure decision processes of domestic agents.

If the definitions of prices used in national accounts and SAMs differ from those used in price driven economic models then there is the potential for contradictions between the economic structures reported in the SAM and those implied by the economic model. Therefore, even if a whole economy model satisfies the conditions of being complete and consistent, but adopts price definitions that do not accord with those used to compile the SAM, there is the potential for incorrect results and/or flawed interpretations of the policy implications.

2.3 Prices and Quantities in a SAM Implicit to a CGE Model

A SAM records transactions that are by definition the product of a price and a quantity, and therefore potentially each price is transaction specific. Assume that LOOP holds so that the price for any transaction in a row is the same irrespective of the agent/account that makes the purchase (see Pyatt, 1987); the subsequent analysis will explain the importance of this assumption and analyse cases where it is apparently breached. This ensures that the quantities in any row are measured in commensurate units and hence they can be meaningfully summed so that the row totals are defined as the product of the respective price and the sum of the quantities that are recorded in each transaction in the row;

$$\sum_j T_{ij} = \sum_j p_i q_{ij} = p_i Q_i \qquad and \qquad Q_i = \sum_j q_{ij} \tag{2}$$

By definition therefore the transactions in each row refer to items that are homogenous.

However, by definition, the total income to an account, i.e., the row total, is identical to the total expenditure by that account, i.e., the column total, and therefore the column total can also be defined as $p_i Q_i$. But while $\sum_i p_i q_{ij} = p_i Q_i$ is meaningful, because the units (transaction values) are commensurate, $\sum_j q_{ij} = Q_j$ is not meaningful because the transaction quantities are not in commensurate units.

Consequently the accounting identities require that the transaction quantities in each row are recorded in commensurate units, i.e., the items are homogenous and hence should have, from an economic logic perspective, a common single price and that each price should be uniquely defined. It is these accounting identities that underpin the LOOP and the price definitions in CGE models (see below).

9 Defined in the SNA as “the amount paid by the purchaser, excluding any deductible VAT or similar deductible tax, in order to take delivery of a unit of a good or service at the time and place required by the purchaser. The *purchaser's* price of a good includes any transport charges paid separately by the purchaser to take delivery at the required time and place” (ISWGNA, 1993, paragraph 215).

3. Prices and Social Accounts[10]

The relationship between price definitions and accounting identities does not derive from some set of bookkeeping/accounting conventions but rather from a series of economic theoretic relationships that were, and still are, inherent to the development of national accounts. These are fundamental behavioural/economic relationships that underpin a large proportion of 'modern' economics and are central to all whole-economy models calibrated with national accounts data.

3.1 Price Definitions and Accounting Identities

Consider the entries in the commodity rows of a SAM, although the same logic applies to all rows since the quantity units in the rows are commensurate. The row totals are the total value of incomes to the commodity accounts, which are the total expenditures on those commodities by agents in the economy, i.e., the total values of demand, and these are equal to the column totals for the commodities, i.e., the total values of supply. In equilibrium total supply equals total demand for each account in both value and quantity terms. Hence the (unique) prices for each row are the average revenues received for each commodity, while 'prices' in the column totals are the average costs. Therefore the accounting identities require that average revenues are defined as identical to average costs, which is consistent, in economic theoretic terms, since costs and prices are codetermined.

This applies to all rows and columns and means that in all CGE models, which conform fully with the LOOP, all prices are derived from accounting identities, whether or not the data are presented as a SAM since the databases for all (consistent) CGE models can be represented as SAMs. Moreover the accounting identities must be observed both *ex ante* and *ex post* since equilibrium solutions require that total incomes and expenditures must equate in equilibrium. Some examples serve to illustrate how this works.

3.1.1 Domestic Price of Imports

Define the domestic price of imports (PM_c) as the world price, *cif*, of imports in domestic currency units plus any import duties. The value of imports at domestic prices is therefore

$$PM_c * QM_c = SAM("row",c) + SAM\left("imptax",c\right) \qquad (3)$$

where QM_c is the quantity of imports, which is a simple accounting identity derived from the SAM. Expressing each SAM transaction as a price multiplied by quantity, making the behavioural assumption that import duties are defined *ad valorem* and simplifying produces

10 The discussion in this section derives largely from Pyatt (1987).

$$\begin{aligned} PM_c * QM_c &= \left(pwm_c * ER * QM_c \right) + \left(pwm_c * ER * QM_c * tm_c \right) \\ PM_c &= pwm_c * \left(1 + tm_c\right) * ER \end{aligned} \quad (4)$$

where pwm_c is the (fixed) world price of imports in foreign currency units, ER the exchange rate and tm_c the *ad valorem* import duty rate. PM_c is an average 'revenue' that is determined by the cost components and, in this formulation, is the *basic* price of imports. Note how the price definition in (4) depends on both the accounting identity and the behavioural assumption about how the tax system operates.[11]

The presumption of *ad valorem* taxes may be neither an appropriate nor a reasonable representation of reality. For instance it may be that the marginal tax rate differs from the average tax rate in which case the simple behavioural relationship in (4) cannot be used although the accounting identity must be preserved. For instance assume that a given quantity of a commodity can be imported at one duty rate but thereafter the import duty rate changes, e.g., a tariff rate quota. This might be expressed as

$$\begin{aligned} PM_c * QM_c &= \left[\left(pwm_c * ER * \overline{QM}_c^1 \right) + \left(pwm_c * ER * \overline{QM}_c^1 * tm_c^1 \right) \right] \\ &\quad + \left[\left(pwm_c * ER * \left(QM_c - \overline{QM}_c^1 \right) \right) + \left(pwm_c * ER * \left(QM_c - \overline{QM}_c^1 \right) * tm_c^2 \right) \right] \\ PM_c &= \left(pwm_c * ER \right) * \left\{ \left[\left(1 + tm_c^1\right) * \frac{\overline{QM}_c^1}{QM_c} \right] + \left[\left(1 + tm_c^2\right) * \left(1 - \frac{\overline{QM}_c^1}{QM_c} \right) \right] \right\} \end{aligned} \quad (5)$$

where $\overline{QM}_c^1$ is the (fixed) quantity imported at the first duty rate (tm_c^1) and tm_c^2 is the other import duty rate. Such a price definition would be appropriate for modelling the UK's corn laws of the 19th century and sugar trade by OECD countries in the 20th century, and can be modelled as mixed complementarity problem where those imports above a defined quantity trigger the implementation of a different regime, e.g., van der Mensbrugghe, *et.al*, (2003).

3.1.2 Basic and Purchaser Prices of Composite Commodities

A common specification in a CGE model, following Armington (1969), is the use of the composite commodity concept whereby domestically produced and imported commodities are aggregated, e.g., Robinson *et al.*, (1990). Hence the *basic* price of the composite commodity (PQS_c) can be specified, using (3) and (4), as

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11 The behavioural assumption of *ad valorem* tax rates has obvious appeal because of the simple form of the price definition equation that it allows. It is also attractive if the solution method uses percentage changes (log differences), as with the MONASH and GTAP models that use GEMPACK; in that class of models it is the percentage changes in the 'power of the tax', $\left(1 + tm_c\right)$, that are solved for rather than the changes in the tax rate. The choice of solution method is not relevant to the derivation of the underlying price definition.

$$\begin{aligned} PQS_c * QQ_c &= SAM(a, c_d) + SAM(\text{"}imptax\text{"}, c) + SAM(\text{"}row\text{"}, c) \\ &= (PD_c * QD_c) + (pwm_c * ER * QM_c) + (pwm_c * ER * QM_c * tm_c) \\ &= (PD_c * QD_c) + (PM_c * QM_c) \end{aligned} \tag{6}$$

where PD_c and QD_c are the price and quantity of the domestically produced commodity sold on the domestic market and QQ_c is the quantity of the composite commodity. This can be written as

$$PQS_c = \left(PD_c * {QD_c}/{QQ_c}\right) + \left(PM_c * {QM_c}/{QQ_c}\right). \tag{7}$$

Hence the *basic* price of the composite commodity is the weighted average of the *basic* prices of its components, i.e., its costs.

In this simple example, with the behavioural assumption of a simple *ad valorem* (general) sales tax and no margins, purchaser prices (PQD_c) are defined as

$$\begin{aligned} PQD_c * QQ_c &= PQS_c * QQ_c + SAM(\text{"}saltax\text{"}, c) \\ &= PQS_c * QQ_c + (PQS_c * QQ_c * ts_c) \\ PQD_c &= PQS_c * (1 + ts_c) \end{aligned} \tag{8}$$

where ts_c is the *ad valorem* sales tax rate, i.e., *basic* prices plus an *ad valorem* sales tax rate. The inclusion of trade and transport margins is a simple extension of the same logic.[12]

It is a common practice to initialise all *basic* prices in a CGE model equal to one and then initialise all other prices relative to *basic* prices using the price definition formulae.[13] Thus in the presence of positive import duties and domestic commodity taxes, the *cif* prices of imports (pwm_c*ER) will be less than one and purchaser prices (PQD_c) will be greater than one.

3.1.3 Purchaser Price of Commodities with VAT

The existence of VAT adds an interesting complication. In a stylised situation all VAT on intermediate inputs would be rebated and hence VAT would only enter into the decision

12 Define an account called 'marg' to carry the domestic trade and transport transactions costs. Now make the behavioural assumption that the trade and transport costs are *ad valorem*, then the price definition can be written as

$$\begin{aligned} PQD_c * QQ_c &= PQS_c * QQ_c + SAM(\text{"}saltax\text{"}, c) + SAM(\text{"}marg\text{"}, c) \\ &= PQS_c * QQ_c + (PQS_c * QQ_c * ts_c) + (PQS_c * QQ_c * mm_c) \\ &= PQS_c + (PQS_c * ts_c) + (PQS_c * mm_c) \\ PQD_c &= PQS_c * (1 + ts_c + mm_c) \end{aligned}$$

where mm_c is the *ad valorem* margin rate. In some models, e.g., the GTAP and GLOBE models, the price definitions for carriage, insurance and freight margins are specified so that the margin rates are based on the quantities of imports rather than the values of imports.

13 It is possible to impose a price normalisation at one of several different stages in the price system. One reason for selecting *basic* prices for the price normalisation is its 'consistency' with the SNA price system.

processes for final demand commodities. Therefore the price paid varies by purchasing agent, which requires different price definitions for intermediate and final demand commodities.

Assume the row labelled '*saltax*' is in fact an *ad valorem* VAT, then there are two price definitions for the purchaser prices of the composite commodity, i.e.,

$$\begin{aligned} PQD_c^a &= PQS_c \\ PQD_c^i &= PQS_c * (1 + tv_c) \end{aligned} \tag{9}$$

where PQD_c^a and PQD_c^i are respectively the intermediate (activity) and final demand (institution) purchaser prices and tv_c is the *ad valorem* VAT rate. The price definitions preserve the economic logic of the LOOP since there is a unique price formation for both *purchaser* prices, even if the SAM representation might suggest only a single price. Importantly the differences in the purchaser prices depend solely on differences in the 'wedges' between *basic* and *purchaser* prices and not on the determination of the *basic* prices.[14] Hence the SAM implicit to the model has commodity accounts that are segmented between intermediate and final demand, in which case the SAM transactions representing the VAT revenues by commodity would have appeared in the column for the commodities sold into final demand. The key point is clearly NOT how the SAM is presented, but knowledge about the price formation process and that the '*saltax*' row in fact refers to a VAT that only applies to final demand, i.e., that the price determination processes were different.

In reality it is unlikely that domestic commodity taxes will only be of one type. In fact it is common to find several different types, e.g., VAT, GST, excise, fuel taxes, etc. Assuming, for simplicity that there are only two taxes – VAT and GST – and that the GST is levied on all commodities sold but that VAT is only levied on final demand commodities and that both are *ad valorem* taxes. Then the price definitions can be written as

$$\begin{aligned} PQD_c^a &= PQS_c * (1 + ts_c) \\ PQD_c^i &= PQS_c * (1 + ts_c + tv_c) \end{aligned} \tag{10}$$

where the calibration of the tax rates defines them as additive.

In a price driven model any misspecification of the price definitions will inevitably distort the results generated by the model and hence potentially lead to inappropriate policy conclusions. Consider the case where the 'true' commodity tax system is that in (9) but it is incorrectly modelled as (8) and reductions in the commodity tax rates (*ts* or *tv*) are imposed. In the modelled situation the costs of production will decline in the first round effect whereas in the 'true' situation they will be unchanged; consequently the price formation processes for the *basic* prices (*PQS*) of domestic commodities will be misrepresented (see below).

14 There is a second order impact of differences in how *purchaser* prices are determined since the prices of intermediate inputs enter into the determination of *basic* prices (see below).

This illustrates the importance of information about the tax system and the need to ensure that the SAM used to calibrate a model provides adequate information. It is also demonstrates that while a SAM may present transactions where the prices are transaction specific, as in equation (1) above, it is necessary effectively to transition from such a format into one in which the LOOP applies, as in equation (2) above.

3.1.4 Production

Consider the case where the SUPPLY matrix in Table 1 is a diagonal square matrix. In this case there will only be one entry on each row, i.e., each activity produces only one commodity and that commodity is unique to that activity.[15] The *basic* price for any commodity produced domestically is then the average revenue for that commodity while the column for the respective activity represents a weighted sum of the costs of producing that commodity, i.e., the average cost inclusive of profits. How the weights operate is a consequence of the behavioural assumptions made about production, i.e., the assumed production functions.

A common assumption in CGE models, e.g., the GTAP model and the USDA ERS model (Robinson *et al*., 1990), is that intermediate inputs are employed in fixed quantities per unit of output – Leontief technology. This means that intermediate input use is insensitive to price changes and allows the specification of a price of value added (PVA_a)[16], subject to the assumption that there is an *ad valorem* production tax, as

$$\begin{aligned} PX_a * QX_a &= \sum_c SAM_{c,a} + \sum_f SAM_{f,a} + SAM_{"prodtax",a} \\ &= \sum_c \left(PQD_c * ioqx_{c,a} * QX_a \right) + \sum_f \left(W_{f,a} * FD_{f,a} \right) + \left(PX_a * QX_a * tx_a \right) \\ &= \sum_c \left(PQD_c * ioqx_{c,a} * QX_a \right) + \left(PVA_a * QX_a \right) + \left(PX_a * QX_a * tx_a \right) \\ PVA_a &= PX_a * \left(1 - tx_a \right) - \sum_c \left(PQD_c * ioqx_{c,a} \right) \end{aligned} \tag{11}$$

where PX_a is the activity output price, QX_a the quantity produced, $ioqx_{c,a}$ are the intermediate input (Leontief) coefficients, $W_{f,a}$ is the price of factor *f* used by activity *a*, $FD_{f,a}$ is the amount of factor *f* employed by activity *a* and *tx* the *ad valorem* production (output) tax rate. Given the definition of the SUPPLY matrix, the *basic* price of the unique commodity produced by each activity, the domestic supply price (PQS_c), is identical to the activity price (for all *a* equal to *c*). As with the commodity price definitions, the activity price is defined by a combination of a behavioural assumption and an accounting identity.

If the *purchaser* price (PQD_c) is specified incorrectly then the production decisions in response to changes in commodity tax rates will be distorted. Assume as before that a GST is

15 The situation where there are multiple product activities is discussed below.

16 This 'trick' is apparently due to Johansen.

imposed instead of a correct VAT system; then a reduction in the domestic commodity tax rate will reduce *PQD*, incorrectly, leading *ceteris paribus* to an increase in *PVA* and thence to increases in factor incomes and so on.

It is trivial to demonstrate that first order conditions for linear homogenous (value added) production functions, e.g., CES and nested CES functions, satisfy the *ex ante* and *ex post* accounting identities. Similarly, an alternative assumption about the production functions that allows for substitution between aggregate primary and aggregate intermediate inputs can be used to derive the price definition for the price of value added, e.g., the IFPRI standard model (Lofgren *et al*., 2001) and the GLOBE model (McDonald *et al*., 2007). While the mechanics may be marginally more complicated, the process of maintaining the accounting identity remains unchanged.

It is common, e.g., in the GTAP model, to find the use of a zero profit condition. The logic of this revolves around ensuring that given the payments to all inputs then the product will be exhausted; this exploits the aggregation properties of linear homogenous functions. While the mechanics are slightly different the principle of aligning behavioural assumptions with *ex ante* and *ex post* accounting identities is unchanged.

3.2 Prices and Interdependence

The relationship between average cost and average revenue also contains another fundamental implication for structural whole economy models that are price driven. Namely, interdependencies within the economic model are determined by the structural information contained within the columns of the SAM, since it is these structural relationships that determine the average costs and hence average revenues, i.e., prices. Consequently it is the relative magnitudes of the entries in the columns of a SAM, i.e., the supply side, that determine the structure of interdependencies and therefore have a fundamental influence upon model performance irrespective of the specific functional forms used in a model (Pyatt, 1987).

This has an important implication for the compilation of a SAM, and by analogy any data structure used for a CGE model; the relevant structural information is contained in the column entries i.e., the Leontief style column coefficients. Thus, when compiling a SAM, it is imperative that an emphasis is placed upon the preservation of the information content of expenditures by accounts; since taxes and subsidies are defined as expenditures and are likely to represent major policy instruments attention to information about taxes and subsidies is clearly critical.[17]

[17] This partially explains why using the RAS method to 'estimate' a new SAM can produce such unreliable results: the RAS method can only preserve the relative magnitudes of the column entries under exceptional circumstances, see Lynch (1979).

4. The 'Law of One Price' and Computable General Equilibrium Models

The relationship between costs and prices provides the economic theoretic reason why LOOP should hold in all consistent CGE models. Indeed LOOP asserts that each set of cost structures should only provide one price because the 'law' is simply a statement of the requirement that each price in the model is uniquely determined. Therefore all CGE models should apparently adhere to LOOP. Furthermore, if any price is not uniquely determined there is an implication that the model contains redundant or misspecified equations. While the universality of LOOP in CGE models seems theoretically self evident it is pertinent to verify this in the context of a number of features commonly found in CGE models.

In common with standard economic theory all CGE models make extensive use of aggregator functions, e.g., utility and production functions, with the most common functional forms being constant elasticity of substitution (CES) and constant elasticity of transformation (CET) functions using both single stage and nested forms.[18],[19] In practice it transpires that CES functions are used when aggregating information contained in the columns of SAMs while CET functions are used when aggregating the information contained in the rows of SAMs. In this section the consistency of these functional forms, as used in CGE models, with LOOP is considered.

4.1 The Armington 'Insight'

Consider first the case of a CGE model that uses the concept of a composite commodity, e.g., models in the tradition of Dervis *et al.*, (1982), and hence a model that is consistent with a SAM structured as in Table 1. In this case an Armington 'function' operates simply as a CES cost function over a specific organisation of the entries in the commodity columns of the SAM. The *basic* price of the composite commodity, PQS_c, is the weighted average of the domestic supply (*basic*) price (PD_c) and the import price (PM_c), and the import price is the world price (pwm_c), in domestic currency units, times one plus the import duty rate (tm_c) (see (4) and (6) above). It therefore accords fully with the arguments for LOOP. The purchaser price (PQD_c) is then straightforward (see (8) to (10) above).

Now consider a case where there are separate commodity accounts for the domestically produced commodities and imports.[20] If the demand for commodities by agents is expressed

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18 Other common forms are translog, Cobb-Douglas and Stone-Geary and combinations of other linear homogenous functions. The arguments developed in the context of CES and CET functions can all be developed using other linear homogenous functions.

19 On the relationships between aggregator (utility, production, Armington, etc.,) functions and their counterpart index number systems, see Diewert (1976). On the properties of nested CES functions see Perroni and Rutherford (1995).

20 The GTAP database is an example of such an arrangement; see McDonald and Thierfelder (2004) for a SAM representation of the GTAP database.

separately for imports and domestically produced commodities then LOOP clearly holds by definition. However it is pertinent to confirm that the LOOP holds if demand is expressed for composite commodities that are aggregates of differentiated imports and domestically produced commodities, e.g., a case where the arguments in household utility functions are composite commodities. If this amounts to a series of CES functions that operate as cost functions over paired sets of commodities then this is no more than a generalised version of the composite commodity case dealt with above: it therefore accords fully with LOOP. The formulation is inevitably slightly more complex if differential 'sales' tax rates apply to imported and domestic variants of a commodity, but such is simply an elaboration.

A formulation with imports that are identical to the domestically produced commodity (homogenous) means that the *basic* price of the commodity for sale to domestic consumers is either the domestic supply (*basic*) price (PD_c) or the import price (PM_c) whichever is lower, i.e., complete specialisation is required. But with complete specialisation the database/SAM will only contain either the production relationships for domestic supply or data on imports; thus, for instance, switching from solely importing a commodity to producing it domestically in a model will be difficult because of the absence of production cost data for the formation of domestic supply prices.

Applications of the Armington 'insight' in CGE models should therefore comply with LOOP; this is the case for models that can be examined in detail. However, both the IFPRI standard (Lofgren *et al*., 2001) and the PEP static models (Decaluwé *et al*., 2009) introduce trade and transport margins in a manner that affects the interpretation of the prices. Specifically domestic trade and transport margins associated with imported commodities are included with the price *PM*, which means that *PM* is not a *basic* price and hence introduces an asymmetry with respect to *PE* that is a *basic* price. Moreover while in both models LOOP is maintained through the calibration process, it is not certain that this will carry over into the compilation of the SAM to calibrate the model since both models use a composite commodity formulation for domestic demand. In particular the specification of different trade and transport margins for imported commodities requires disaggregated data for the supply and demand for imports that are not included in the SAM used to calibrate the models.

4.2 Export Transformation Functions vv Homogenous Exports

Typically a SAM is presented with export transactions (foreign demand for commodities) appearing in the same (commodity) rows as domestic demand, which suggests that for LOOP to hold the export price must be the same as the domestic price. This is the layout in Table 1 above, but closer analyses of the behavioural relationships underpinning models with transformation (CET) functions for exports, e.g., Dervis *et al*., (1984), Lofgren *et al*., (2001), van der Mensbrugge, (2005) and McDonald *et al*., (2007), demonstrates that this is a potentially misleading, if parsimonious, SAM presentation of the data that obscures the behavioural assumptions.

The use of an export transformation function involves the assumption that each activity actually produces two variants of the 'single' commodity that are differentiated by destination - export and domestic commodities – and hence have different prices. A standard behavioural assumption is that each activity produces a composite commodity (QXC_c) using the single technology represented by the activity specific (composite) production function, and then this composite commodity is differentiated according to destination. The price definition process then varies with respect to the output mix between domestic and export commodities, i.e.,

$$PXC_c = \left(PD_c * {QD_c}/{QXC_c} \right) + \left(PE_c * {QE_c}/{QXC_c} \right) \tag{13}$$

where PXC_c is the *basic* price of the composite commodity, and PE_c and QE_c are the *basic* price and quantity of exports. But the *basic* price of the composite commodity is determined by the cost structure of the sector that produces it and the cost structure is invariant to the output mix.

This sidesteps the requirement for two activity accounts by allowing the use of a single (composite) production function for both the domestic and export commodities, but involves the (hidden) presumption that although the commodities are differentiated the production functions are identical.

Hence there is a tension in any CGE model that includes a CET formulation for modelling the assumption that the domestic and export commodities are differentiated. Namely despite the presumption of differentiation the commodities are produced with identical technologies **AND** the input mixes of the relevant activities do **NOT** change as the output mix changes.

It is interesting to assess the difference between a model that assumes the export and domestic commodities are differentiated and one in which they are homogenous. The key difference is that the requirement for two 'different' activity accounts is no longer necessary. Since the export and domestic commodities are homogenous they are priced identically ($PE_c \equiv PXC_c \equiv PD_c$) it is only necessary to record two types of commodity account - one for the domestic commodity and imports and the other for exports. The separate export account is required because of the possibility that the export and domestic purchaser prices might differ despite a common *basic* price due to differences in sales and export tax rates and margins; the distinction by agents is in effect implemented on the supply side of the model. Thus whether it is assumed that the domestic and export commodity are homogenous or differentiated does not alter the implied requirement for two sets of commodity accounts; one for the domestic commodity and another for exports. While the presumption of homogeneity resolves the tension by ensuring unique price formation processes for the domestic and export commodities it does so at a cost: imports are treated as heterogeneous while exports are treated as homogenous.

Consequently all models that include CET functions for the modelling of export supply include a violation of LOOP, e.g., the IFPRI standard, PEP static, LINKAGE and GLOBE models, since differentiated commodities are produced using a single technology, i.e., cost structure. The GTAP model (Hertel and McDougall, 2003) is a notable exception in the literature since it assumes the domestically produced commodities are homogenous irrespective of destination market. This raises the issue of why LOOP is breached in so many models. Overwhelmingly it seems that LOOP is breached because the assumption of homogenous commodities sold on domestic and export markets requires the presumption that agents do not respond to relative price changes on domestic and export markets, which may be regarded as anomalous behaviour, and, certainly in global models, can produce terms of trade effects that are deemed excessively large.[21] The extent to which the use of CET functions for export supply and the breaching of LOOP are understood is not clear in the literature. What is clear is that while commodities that are exported and consumed domestically are typically recorded in the same rows they are not modelled as such in CGE models with or without CET functions for export supply.[22]

4.3 Multi-Product Activities

Consider the case where each activity can produce multiple different commodities and each commodity can be produced by multiple activities; this would be the case where a SUPPLY and USE SAM was used to calibrate the model, which is the case in the IFPRI standard model (Lofgren *et al*., 2001) and the PROVIDE model (McDonald, 2005). Such a formulation breaks the unique linkages between the activity and commodity prices that were used above (section 3.1.4) to link production costs to commodity prices.

The case of multiple domestic sources (activities) for a commodity is relatively straightforward. The typical assumption is that commodities are differentiated by the activity that produces them and then a composite produced commodity can be defined as a CES aggregate of the activity differentiated varieties of that commodity. This is simply an application of the Armington ‘insight’ to domestically differentiated commodities, and therefore is not problematic with respect to LOOP.[23]

21 In the GTAP model these large terms of trade effects are ‘damped’ down by high elasticities of substitution for import demand.

22 Note that if prices are normalised on *basic* prices this difference is masked.

23 Where varieties of a commodity are perfect substitutes is a trivial extension.

Much more interesting is the treatment of multiple commodities produced by one activity since there is only one production function – input cost structure – that provides the price determination process for multiple commodities. The IFPRI standard model (Lofgren, *et al*., 2001) adopts the assumption that the products are produced in fixed proportions; this means that the (implicit) assumption of a composite production function is not ‘stretched’ by allowing the output mix to change without any change in the input structure. The *de facto* assumption underpinning this approach is that the non characteristic commodities are produced as by-products of the characteristic commodity of the activity. Combined with the assumption of CES aggregation across varieties of a commodity the *ex post* accounting identity is maintained by adjustments to the average revenue.

The by-product assumption in the IFPRI standard (Lofgren et al., 2001) and PROVIDE models (McDonald, 2005) seems reasonable in circumstances where the proportions of the principal product of an activity are very large. However in circumstances where the outputs of activities are strongly diversified the by-product assumption can produce perverse results, e.g., the output of a commodity can increase substantially even if its *basic* price falls because the overall output of an activity, or group of activities, increases due to price rises for other commodities. This situation most commonly arises for agricultural activities and commodities when the activities are defined by location, with fixed areas of land, while commodities are defined conventionally. Then the by-products/fixed proportions assumption implies that farmers are not responsive to output price changes in their production decisions. This is arguably an indefensible assumption, which can also produce unrealistic results.

Several models have sought to counteract such perverse results by aggregating activity outputs using CET functions, e.g., Gohin and Meyers (2002), a version of the PROVIDE model (McDonald and Punt, 2007) and the PEP model (Decaluwé *et al*., 2009) [24]. By doing so LOOP is clearly breached since the output mixes of activities are allowed to vary but it is assumed that this happens without any changes in cost structures. Again the method appears to be a pragmatic solution to a problem of perverse results, but it is unclear to what extent the implications of breaching LOOP are recognised. For instance, CET aggregations across multi product activities will have different second ‘round’ effects because changes in each activity’s output mix it will impact differently upon the domestic demand for the outputs of other activities.

Consequently any CGE model that is calibrated using data for multi-product activities places some strain upon the specification of the production functions. This makes it tempting to argue a case of using input-output tables rather than SUPPLY and USE tables in the SAM, but since input-output tables are conventionally derived as linear transformations of supply and

24 It has not been possible to verify this model due to lack of access to the code.

use tables (see Miller and Blair, 1985), the difficulties are not resolved, rather they are bypassed (assumed away) and then hidden.

4.4 Aggregation of Primary Factors

As with multi-product activities the aggregation of primary factors needs to operate across both the columns and rows of the SAM. The aggregation across primary factors used by an activity is straightforward since it is an application of standard, e.g., CES, production functions whereby the first-order conditions ensure a factor is paid the value of its marginal product in its use by that activity.

But an aggregation rule for primary factors is required to achieve equilibrium in factor markets, except in the extreme special cases where factor use is either activity specific and immobile or where there is excess supply of all possible factors. If LOOP holds then each identified factor is required to be homogeneous, i.e., have an identical value of marginal product in all activities, and this is the underlying assumption when factor quantities are unknown and hence values and quantities are, effectively, equated by normalising the factor prices. This is the situation in all models calibrated from the GTAP database. However this is an extreme assumption that breaks down when factor quantities are known and when the assumption of fully employed and fully mobile factors is relaxed. Thus in the vast majority of practical CGE models LOOP typically holds only in the presence of imperfect information – no knowledge of factor quantities – and even then is violated if the full employment and/or full mobility assumptions are relaxed.

Models in the tradition of Dervis *et al*., (1982), e.g., the IFPRI standard and GLOBE models, achieve factor aggregation by imposing the assumption that the marginal productivity of each factor type is conditioned by activity specific factor productivity parameters.[25] In the full factor employment and mobility case a factor can move between activities and still be aggregated in ‘natural’ units so as to satisfy factor market clearing conditions. If the full employment and full mobility assumptions are relaxed then factors can still be aggregated in ‘natural’ units so as to satisfy factor market clearing conditions by allowing the activity specific factor productivity parameters to vary. However this approach can produce perverse results when factor quantities are known and the marginal productivities of the ‘same’ factor vary widely across activities; in such cases the factor reallocations due to policy shocks can produce very large changes in the ‘average’ productivities of factors that dominate the results and are arguably implausible. The PROVIDE database for South Africa has this property (PROVIDE, 2006).

25 In terms of equation (12) this approach means that $W_{f,a} = WF_f * WFDIST_{f,a}$ where WF_f is an ‘average’ factor price across all activities and $WFDIST_{f,a}$ the activity specific adjustment. $WFDIST_{f,a}$ are typically parameters but can be allowed to be variables when factor market clearing conditions other than full employment and mobility are sought.

Alternatively a CET aggregation across differentiated variants of the same factor might be implemented, i.e., an aggregation across a row of a SAM. This is instinctively appealing for land where, for instance, pasture and arable land may be substitutable but the marginal productivities in different uses diverge. The GTAP (Hertel and McDougall, 2003) and the GTAP-BIO models (Birur *et al*., 2008) use CET functions to aggregate primary inputs across different activities so as to define factor market clearing conditions. But one aspect of such an aggregation method is that the total quantities of a factor in natural and efficiency units vary with the patterns of use of that factor by different activities, i.e., there is no longer a uniform relationship between the stock of and flow of services from a factor, hence the definition of market clearing for that factor may be opaque. While the use of CET functions in such cases may be justified on the grounds that the factors are differentiated but there is a lack of data. Although this may be a pragmatic solution it does mask the underlying issue, which is the absence of data underpinning the SAM.

4.6 Agent Specific Commodity Taxes

From an accounting perspective there is no absolute reason why all transactions in a row of a SAM must be valued at the same price; for instance the sales taxes in Table 1 may be the summation of a number of different commodity tax instruments – GST, excise taxes, fuel taxes – that are levied at different rates, or it may be that markets are segmented such that different agents pay different GST rates on purchases of the 'same' commodity. The most general case is where all agents make purchases on which different sales/commodity tax rates can apply. In such a case the simple segmentation between GST and VAT developed in section 3.1.3 is not sufficient and a more general formulation is required.

The GTAP model (Hertel and McDougall, 2003), and other models in the ORANI tradition, are examples of CGE models that allow agent specific commodity taxes. The SAM representation of the GTAP database (McDonald and Thierfelder, 2004)[26] provides one method for recording agent specific commodity taxes: the entries in the domestic commodity rows are valued at *basic*[27] prices and agents pay sales taxes direct to the government that are paired with the purchases of commodities.[28]

In the GTAP model the effective price paid by agents for domestic commodities, ignoring the region subscripts, can be defined, from the column accounting identities, as

26 The SAM representation of the GTAP database is, since 2010, available as part of the GTAPAgg software package with which the database is distributed.

27 GTAP does not use the SNA terminology for prices. Instead it defines prices as agent, market and world prices which differ somewhat from SNA prices.

28 The recording of exports in the domestic commodity rows has been dealt with above.

$$PDD_{c,sac}^{g} = PXC_{c}^{g} * \left(1 + ts_{c,sac}^{g}\right) \qquad (13)$$

where *sac* is the set for all agents in the model, $PDD_{c,sac}^{g}$ is the GTAP *purchaser* price for purchasing agent *sac* of the domestically produced commodity *c*, PXC_{c}^{g} is the GTAP *basic* price of the domestically produced commodity *c* and $ts_{c,sac}^{g}$ is the specific commodity tax for domestically produced commodity *c* and purchasing agent, *sac*. Hence the derivation of *purchaser* prices in the GTAP model is fully consistent with LOOP; namely that of unique price determination. Arguably this is an approach that is economically correct since it ensures LOOP holds in all cases; provided the necessary data are available.

An alternative presentation of the GTAP database in SAM format would segment each commodity account so that the rows, and columns, for the commodity accounts were categorised by both commodity and purchasing agent. Such a method would mean that the sales tax entries in the respective columns identified the sales taxes paid by each agent on each commodity and the entries in the rows would be valued in *purchaser* prices, and LOOP would apply to the rows of the SAM. Either presentation serves the same purpose: the recording of the data in the SAM in such a manner as to ensure that the information content of the SAM identifies the costs that determine prices.

5 Home Production for Home Consumption (HPHC)[29]

National account statisticians are expected to, and do, include imputations for HPHC subject to the limitation that imputed HPHC only includes commodities that are produced within the SNA's production boundary, i.e., commodities for which there are clearly identifiable marketed equivalents so that unique prices can be assigned to the commodities so produced and consumed (see ISWGNA, 1993, paragraphs 6.17 and 6.18)[30].

If HPHC represents a sufficiently small part of economic activities within the production boundary then, arguably, the separate identification of HPHC may not be important. But for many developing countries, especially those where semi subsistence agriculture remains an important source of livelihoods, HPHC is a very important form of economic activity.[31] Since the price formation processes for HPHC may differ greatly from those for marketed

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29 Lofgren *et al*., (2001) and some SAMs configured to work with the IFPRI standard model include a component in the model and sub matrix of data designed to capture HPHC. The argument developed below implicitly questions the approach used in that model and associated SAMs; but it is important to recognise that that model is one of only two that are known that seek to encompass HPHC.

30 This means that domestic services - catering, child rearing, and carer services - that fall out with the SNA production boundary are not included in measured national product. This is not an assertion that such activities are unimportant; rather that the units of measurement are inadequately defined to ensure appropriate measurement.

31 In Ethiopia some 90 percent of the labour force are either self employed or engaged in 'unpaid family labour', while some 80 percent of the labour force are engaged in some form of semi subsistence agriculture where HPHC accounts for a very large share of the VALUE - note that the prices of commodities that are home produced and consumed are very much lower than similar commodities purchased in the market and therefore the 'volume' share of HPHC is even more pronounced - of consumption by households.

commodities a decision to ignore HPHC in the data and model formulation is a potentially large source of bias in the results from quantitative analyses, e.g., liberalisation of agricultural trade policies may have limited impacts on agricultural households that sell little of their output.

Table 2 provides an illustration of a SAM organised so as to identify the differences in price formation processes associated with HPHC. The relevant defining features of HPHC are

1. HPHC consumed commodities do not enter the market and hence the price formation processes do not include taxes or marketing margins;
2. HPHC commodities can only be consumed by the household that produced them;
3. RHGs that engage in HPHC have (paired) accounts as both activities and households;
4. HPHC activities are likely to be multi product activities; and
5. the resources of RHGs used in HPHC cannot be sold onto factor markets.

Semi subsistence[32] agriculture accounts for a large proportion of HPHC and will be used as the example. In the simplest case farm households engaged in semi subsistence agriculture sell part of the output of their activities on the market, in which case it enters the marketing system, while the rest of their output is consumed only by the household that produced the output. HPHC and marketed outputs are sold at *basic* prices, following the SNA convention on prices within the production boundary. Thus each agricultural commodity would have TWO accounts; one for the commodity that is HPHC, demand for which is valued in *basic* prices since it incurs no commodity taxes or margins, and one for the commodity that is purchased on the market and therefore valued at *purchaser* prices. Thus in Table 2 HPHC activities are reported as receiving incomes in two sub matrices; the first, HPHC activities/HPHC commodities, records commodities that are produced and consumed at home while the second, HPHC activities/Market commodities, records sales to the market.

The only source of final demand for HPHC (non marketed) commodities is the RHG; therefore each representative household group (RHG), that engages in any HPHC, must be paired with a HPHC activity: the RHG is the destination for final demand while the paired account for the HPHC activity is the destination for intermediate demands, and the HPHC activity is the sole source of supply. The ‘correct’ prices for valuing HPHC are the *basic* prices received for the same commodities sold by the HPHC activity to the market.

Derivation of these *basic* prices requires the specification of the input/cost structures for each HPHC activity, i.e., the column entries for the HPHC activity accounts. Since each HPHC activity can have different costs of production the *basic* prices for each HPHC commodity can

32 Absolute self-sufficiency may be an interesting abstraction, but it is only a special case of semi subsistence agriculture and does not need a different treatment.

differ for each household. This indicates that the number of columns and rows for HPHC commodities could equal the number of HPHC activities multiplied by the number of commodities that are HPHC; but since each RHG can only consume the output of the HPHC activity with which it is paired there, should be no confusion in a model since this condition will have to be imposed in the model's behavioural relationships.

Table 2 A SAM with Home Production for Home Consumption (about here)

Assuming a SAM so configured can be produced it is interesting to consider how this might be used in a CGE model; in particular it is important to model the demand for and supply of HPHC commodities, the market clearing conditions for commodities and factors, and the implications for income distribution and expenditure.

The demand side is relatively straightforward. Each RHG can consume a range of commodities amongst which some may be the 'same' commodity sourced from HPHC and marketed sources, which implies the need to modify the utility functions. Assume a standard utility function is used, e.g., Stone-Geary, then it is arguably inappropriate to assume that the substitutability between a commodity differentiated by whether it is marketed or HPHC is on a par with different marketed commodities. This can be resolved by creating composite commodities from the HPHC variants, via CES aggregates, and then these composite commodities can enter as arguments in the utility functions of the RHG. Thus demand can be modelled by nested utility functions, which is simply a standard method of (column) aggregation in CGE models and is consistent with LOOP.

The supply of HPHC commodities is marginally more complex. Since each RHG must have a matching activity the information exists to define the cost structure of production using a standard production function approach. Market clearing conditions for the HPHC commodities are then straightforward; the only source of demand is the paired RHG so market clearing requires that the amount of HPHC supplied by activity the must equal the amount demanded by the household with the activity supplying this before selling output to the market. But not only do household production activities have to segment their output between HPHC and the market they, almost by definition, engage in multi-product production processes. Hence the issues associated with changes in the cost structure of multi-product activities discussed above (see section 4.3) arise. Thus pragmatic considerations may induce modellers to breach LOOP.

Factor market clearing is more complex since it also requires resolving time allocation problems for the RHGs. At the RHG level factor market clearing requires that the supply of factors by each RHG must be greater than or equal to the demand for those factors, and this places an upper limit on the factors owned by a RHG that can be used by its paired activity, but not on the total quantities of factors that can be used by that activity, i.e., (marketed) factors can be hired from other RHGs. Thus not only must factor market clearing be realised at the RHG level, total supply greater than or equal to total demand, it is also necessary to impose the constraint that total factor demands for own HPHC are less than or equal to the supply from the RHG. If HPHC does not exhaust the RHG supply of factors then these 'surplus' factors can be sold to other activities. This means that the income to each RHG can consist of factor incomes derived from HPHC, which are totally absorbed by the RHG's demand for HPHC commodities, factor incomes derived from the production of marketed commodities and factor incomes from the sale of factor services to other activities. The demand for factors encompasses activities within and without the production boundary; clearly the supplies of factors available from RHGs within the boundary depend in part on activities that take place outwith the boundary and are exogenous to this model.

The rates of return to factors used in HPHC activities are less clear and made less so by the fact that the alternative may be unemployment. Payments to hired factors should be recorded so the problem simplifies to the returns to own (RHG) factors, which raises the classical problem avoided in national accounts by using a mixed income account; specifically limited information about factor use in (semi-subsistence) agriculture. But such an account is not viable in a CGE model and therefore indicates a need for imputed factor prices. Total returns to factors are known, i.e., the prices of value added defined above, so the issue is to distribute returns across own factors. The standard approach of using activity specific marginal productivities for defined labour types could be used but this requires finding a resolution to the classification of labour types, assuming either perfect or no mobility and accepting the possibility of large changes in factor productivities. An alternative approach is to assume capital and land receive the prices they receive in other (similar) activities and then to define a HPHC category of labour that receives the residual. Since the HPHC labour cannot be used by other activities the mobility of this type of labour across activities cannot be modelled in the traditional manner. One alternative is to use migration functions so that HPHC labour moves across categories of labour in response to changes in relative wage rate, a partial adjustment response (see McDonald and Thierfelder, 2008, for an example of this approach).

6. Concluding Comments

Not only must the database used to calibrate whole economy models be complete and consistent so to must be the database that can be derived from the model solution; i.e., the accounting identities for all agents must be satisfied both *ex ante* and *ex post*. And since the prices (average revenues) for each and every account are derived from the input structures and input costs (prices) for the respective accounts, it is a requirement for model consistency that the price definitions – or linkages – must be derived from the input structures and costs (prices). Consequently the 'law' of one price is not just an accounting convention for a SAM; rather in modeling terms it is an economic theoretic requirement that each and every price is uniquely determined. Thus an understanding of the economic theoretic basis for LOOP is important both to the construction of databases for whole economy models and for the use of those databases to calibrate whole economy models.

The implementation of this approach to price definitions is simple when dealing with price definitions that depend solely upon entries in the columns of a SAM, whether this is by linear homogeneous (CES) aggregation or by the implementation of (tax and/or margin) wedges between pairs of prices. The discussion has however shown that the common use of (liner homogenous) transformation (CET) aggregations does strain the requirement that prices are uniquely determined since typically one set of input structures and costs can generate more than one price. The implicit assumption underlying such transformation functions is that there is some form of segmentation on the supply side that preserves the price differences. Moreover, the use of CET aggregations does create difficulties in defining market clearing conditions when factors can be measured in natural units.

Thus if CET functions are used for row aggregations in CGE models it is incumbent upon the modeller to both justify the decisions to violate LOOP and to demonstrate that the pragmatic advantages derived by violating LOOP outweigh the disadvantages. Generally it is reasonable to conclude that using CET functions simply to avoid collecting data is indefensible, whereas using CET functions to compensate for unavailable information and/or to ensure behaviours by agents that are more realistic may be defensible.

The apparent separation of data and model that is implicit to many whole economy models has many useful features, but suffers from several problems that are potentially serious. Not the least of these problems is the inclination to assume that the national accounts data used by these models can be safely left in the hands of national accounts statisticians and neglected by the economists who carry out the modelling exercises. The arguments advanced in this paper indicate that neglect of the national accounting conventions by modellers may be a mistake that will compromise both the quality of the data used to calibrate whole economy models and the quality of the whole economy models. It is argued that economic modellers would be well advised to remember that national accounts were originally devised for, among other purposes, the implementation of quantitative economic models and consequently national accounts conventions contain well specified systems of prices. An understanding of these systems is clearly important to the specification of the price formation processes in price driven economic models. If economic models and national accounts are to remain complementary activities it is argued that economists need to reengage with national account statisticians: it is a tribute to the enduring legacy of Richard Stone “that others have not had to reinvent the architecture of the national accounts” (Pyatt, 2005) despite the arguably relative neglect by economists.

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Table 1 Structure of a SAM

Incomes	Expenditures	1	2	3 Factors		4 Institutions			5	6	7
		Commodities	Activities	Labour	Capital	Households	Enterprises	Government	Capital Account	Rest of World	Total
1	Commodities		Intermediate inputs (Combined USE)			Households consumption		Government consumption	Investment	Exports (*fob*)	Total demand
2	Activities	Domestic production (SUPPLY)									Production
3	Factors										
	Labour		Wages							Factor Incomes from abroad	GNP at factor cost
	Capital		Profits & Rent							Factor Incomes from abroad	
4	Institutions										
	Households			Labour income	Distributed profits	Intra-household transfers		Transfers		Transfers from abroad	Household income
	Enterprises				Non-distributed profits		Transfers	Transfers		Transfers from abroad	Firms income
	Government	Tariffs; VAT; Export taxes, Sales taxes	Production taxes	National insurance	Distributed profits; Taxes on profits	Direct taxes	Direct taxes			Transfers from abroad	Government income
5	Capital Account					Household saving	Firms saving	Government saving		Capital transfers	Total savings
6	Rest of World	Imports (*cif*)		Factor payments			Current transfers abroad				Imports
7	Total	Total supply	Production	Factor outlay		Households expenditures	Firms expenditures	Government expenditures	Total investment	Foreign earnings	

Table 2 A SAM with Home Production for Home Consumption

Incomes	Expenditures	1a	1b	2a	2b	3		4			5	6	7
						Factors			Institutions				
		HPHC Commodities	Market Commodities	HPHC Activities	Activities	Labour	Capital	Households	Firms	Government	Capital Account	Rest of World	Total
1a	HPHC Commodities			HPHC intermediate inputs				HPH consumption					Total HPHC demand
1b	Market Commodities			Intermediate inputs	Intermediate inputs			Household consumption		Government consumption	Investment	Exports (fob)	Total Market demand
2a	HPHC Activities	Non marketed HPHC	Marketed HPHC										HPHC Production
2b	Activities		Domestic Production										Market Production
3	Factors												
	Labour			Imputed Wages	Wages							Factor Incomes from abroad	GNP at factor cost
	Capital			Imputed Profits & Rent	Profits & Rent							Factor Incomes from abroad	
4	Institutions												
	Households					Labour income	Distributed profits	Intra-household transfers		Transfers		Transfers from abroad	Household income
	Firms						Non-distributed profits		Transfers	Transfers		Transfers from abroad	Firms income
	Government		Tariffs; VAT; Export taxes, Sales taxes		Production taxes	National insurance	Distributed profits; Taxes on profits	Direct taxes	Direct taxes			Transfers from abroad	Government income
5	Capital Account							Household saving	Firms saving	Government saving		Capital transfers	Total savings
6	Rest of World		Imports (cif)			Factor payments			Current transfers abroad				Imports
7	Total	Total HPHC supply	Total Market Supply	HPHC Production	Production	Factor outlay		Households expenditures	Firms expenditures	Government expenditures	Total investment	Foreign earnings	