





# R23 Model: A SAM Based Global Macro CGE Model

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#### **Abstract**

This paper provides a technical description of a global computable general equilibrium (CGE) model calibrated from a global Social Accounting Matrix (SAM) of 203 countries. The database extends the Global Trade Analysis Project (GTAP) database with additional current account data – remittances, aid and foreign factor income. The R23 database emphasises macroeconomic linkages between countries and therefore has less information about the structure of production than does the GTAP database. The model has nominal and real exchange rates, a 'dummy' region, Globe, for recording inter-regional transactions where the source or destination are unidentified, and relationships governing remittances, aid transfers and payments for foreign-owned factors.

*Keywords*: Computable General Equilibrium; foreign aid; remittances.

JEL classification: D58; R13; F49.

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# Introduction

This paper provides a technical description of the R23 model, which is a multi-region (global) variant of the 123 model (de Melo and Robinson, 1989; Devarajan, et al., 1990) where the global dimension is based on the modelling of global trade relations used in the GLOBE model (McDonald et al., 2007 and 2013). The R23 model is calibrated with a database presented in the form of global Social Accounting Matrix (SAM); this variant uses data from the R23 database, which has globally complete and consistent macroeconomic SAMs for 203 countries. The underlying data are from the World Bank's World Development Indicators (WDI) data base; the IMF's Balance of Payments Statistics (BOPS) database; the IMF's Government Financial Statistics (GFS) database; the OECD's Development Assistance Committee (DAC) database containing data on aid payments made by DAC members; and the GTAP 8 database. <sup>2</sup> The R23 database emphasizes macroeconomic linkages among countries, including information on bilateral remittances; bilateral payments for workers compensation; bilateral trade flows; bilateral foreign factor income flows; bilateral and multilateral aid flows, the latter being aid transfer effected through international institutions, e.g., development banks, etc. The model owes a lot to the development of the SAM approach to national accounting, e.g., Stone (1962a and b) and Pyatt (1991), and the SAM approach to modelling, e.g., Pyatt (1987), Drud et al., (1986).<sup>3</sup>

The underlying approach to multi-region modelling for the R23 model is the construction of a series of single country 123 models that are linked through their trading relationships.<sup>4</sup> The price systems in the model are linear homogenous and hence the focus is upon movements in relative, rather than absolute, prices. Consequently, each region in the model has its own numéraire price, typically the consumer price index (CPI), and a nominal exchange rate, while the model as a whole requires a numéraire, which is an exchange rate

See McDonald *et al.* (2015) for a detailed description of the derivation of the R23 database.

While the R23 model is a member of a family of CGE models that model trade relationships using principles described in the 1-2-3 model, features in this model stem from other developments in CGE modelling. Among the models that have influenced this model are the US Department of Agriculture model (Robinson *et al.*, 1990; Kilkenny, 1991) and NAFTA models (Robinson *et al.*, 1993), the IFPRI standard model (Lofgren *et al.*, 2002), the PROVIDE Project model (McDonald, 2003), and the GTAP model (Hertel, 1997). The model owes substantial debts to the wider community of CGE modellers.

A version of the 123 model that uses an aggregation of the R23 database is available (McDonald, *et. al.*, 2014).

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index for a number of reference regions, i.e., there are (r + 1) numéraires. As such this model contains a fundamentally different philosophical approach to global modelling to that found in the GTAP model.<sup>5</sup> Behind this difference lies a theoretical debate about how comparative static and finite horizon dynamic CGE models should value transfers associated with the capital account of the balance of payments (see Robinson, 2004).

A distinctive feature of the model is the use of a 'dummy' region, known as Globe, that allows for the recording of inter-regional transactions where either the source or destination regions are not identified. Examples of such transactions include trade and transportation margins and multilateral aid transfers. The Globe construct provides a general method for dealing with any transactions data where full bilateral information is missing.

The rest of this paper is organised as follows. Section 2 reviews the data used in the model; this section also provides a brief description of how the data were compiled into a SAM. This is followed in section 3 by a descriptive overview of the model and then, in section 4, by a formal description of the model's equations. The description in section 4 is based upon a default setting for the model closure rules; one of the model's key features is the flexibility of the closure rules and consequently section 5 considers the alternatives built into the model's basic structure. This is followed by some concluding comments that primarily focus upon possible model developments. Finally, there is a companion User Guide for the R23 model that provides guidance on using the R23 model and information about various ancillary and supporting programmes that have been developed.

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Variants of the R23 and GLOBE models exist that replicate the (r + 1) numéraires used in the GTAP model are replicated (McDonald *et al.*, 2016).

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# **Model Data**

The R23 database used by the model are presented in a three dimensional Social Accounting Matrix (SAM). There is information on incomes/production and expenditures/consumption by (aggregate) agents for each of the 204<sup>6</sup> countries in the data base. The countries are linked by bilateral trade flows, bilateral remittances, bilateral worker compensation payments, bilateral foreign factor income flows, and bilateral and multilateral aid payments.

Detailed descriptions of the data and methods for developing the SAM database are provided in a companion document (McDonald *et al.*, 2015). The discussion here is limited to the general principles.

# 2.1 Global Macro Social Accounting Matrix

A SAM is a transactions matrix; hence each cell in a SAM simply records the values of the transactions between the two agents identified by the row and column accounts. A description of the transactions recorded in a representative SAM for a typical region in the database is provided in Table 2.1. The selling agents are identified by the rows, i.e., the row entries record the incomes received by the identified agent, while the purchasing agents are identified by the columns, i.e., the column entries record the expenditures made by agents. As such a SAM is a relatively compact form of single entry bookkeeping that is complete and consistent and can be used to present the National Accounts of a country in a single two-dimensional matrix (see UN, 1993, for a detailed explanation of the relationship between conventional and SAM presentations of National Accounts). A SAM is complete in the sense that the SAM should record ALL the transactions within the production boundary of the National Accounts, and consistent in the sense that income transactions by each and every agent are exactly matched by expenditure transactions of other agents. A fundamental consequence of these conditions is that the row and column totals of the SAM for each region must be identical, and hence the SAM provides a complete characterisation of current account transactions of an economy as a circular (flow) system.

<sup>6 203</sup> individual countries and one Rest of the World region.

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Table 2.1 Macro Social Accounting Matrix for a Region in the Global Macro Social Accounting Matrix

|                  | Commodities                                  | Activities                                       | Factors                          | Households                     | Government                      | Capital                   | Margins                             | Rest of<br>World                  | Totals                               |
|------------------|--|--|----------------------------------|--------------------------------|---------------------------------|---------------------------|-------------------------------------|-----------------------------------|--------------------------------------|
| Commodities      | 0  | Combined<br>Intermediate Use<br>Matrix           | 0                                | Private<br>Consumption         | Government<br>Consumption       | Investment<br>Consumption | Exports of Margin<br>Services (fob) | Exports of Commodities (fob)      | Total Demand for<br>Commodities      |
| Activities       | Domestic Supply<br>Matrix                    | 0  | 0                                | 0                              | 0                               | 0                         | 0                                   | 0                                 | Total Domestic<br>Supply by Activity |
| Factors          | 0  | Expenditure on Primary Inputs                    | 0                                | 0                              | 0                               | 0                         | 0                                   | Factor incomes from other regions | Total Factor<br>Income               |
| Households       | 0  | 0  | Distribution of Factor Incomes   | 0                              | 0                               | 0                         | 0                                   | Remittance income                 | Total Household<br>Income            |
| Government       | Taxes on<br>Commodities                      | Taxes on<br>Production<br>Taxes on Factor<br>Use | Direct/Income<br>Taxes           | Direct/Income<br>Taxes         | 0                               | 0                         | 0                                   | Aid incomes                       | Total Government<br>Income           |
| Capital          | 0  | 0  | Depreciation<br>Allowances       | Household Savings              | Government<br>Savings           | 0                         | Balance on<br>Margins Trade         | 'Current account' balance         | Total Savings                        |
| Margins          | Imports of Trade<br>and Transport<br>Margins | 0  | 0                                | 0                              | 0                               | 0                         | 0                                   | 0                                 | Total Income from<br>Margin Imports  |
| Rest of<br>World | Imports of Commodities (fob)                 | 0  | Factor payments to other regions | Remittance expenditures        | Aid expenditures                | 0                         | 0                                   | 0                                 | Total Income from<br>Imports         |
| Totals           | Total Supply of<br>Commodities               | Total Expenditure<br>on Inputs by<br>Activities  | Total Factor<br>Expenditure      | Total Household<br>Expenditure | Total Government<br>Expenditure | Total Investment          | Total Expenditure on Margin Exports | Total Expenditure on Exports      |                                      |



A global SAM is then simply a three-dimensional matrix where the first and second dimensions identify the incomes and expenditures by agents within a country and the third dimension identifies the country/region. For a global SAM the complete and consistent conditions need extending to encompass transactions between regions; this simply requires that each and every import transaction by a region must have an identical counterpart export transaction by another region. This is enough to ensure that the resultant global SAM provides a characterisation of current account transactions of the global economy as a circular (flow) system.

Given these definitions of a SAM the transactions recorded in a SAM are easily interpreted. In Table 2.1 the row entries for the commodity accounts are the values of commodity sales to the agents identified in the columns, i.e., intermediate inputs are purchased by activities (industries etc.), final consumption is provided by households, the government and investment demand and export demand is provided by the all the other regions in the global SAM and the export of margin services. The commodity column entries deal with the supply side, i.e., they identify the accounts from which commodities are purchased so as to satisfy demand. Specifically, commodities can be purchased from either domestic activities – the domestic supply matrix valued inclusive of domestic trade and transport margins – or they can be imported – valued exclusive of international trade and transport margins. In addition to payments to the producing agents – domestic or foreign – the commodity accounts need to make expenditures with respect to the trade and transport services needed to import the commodities and any commodity specific taxes.

The R23 database, following the lead of the GTAP database, provides complete coverage of bilateral transactions in commodities – these are valued free on board (fob) - but only provides partial coverage of transactions in trade and transport margins. Specifically, the imports of trade and transport margins by each region are associated directly with the imports of specific commodities, hence for commodity imports valued fob the source and destination regions are identified and the value of trade and transport (margin) services used are identified. The sum of the values of trade and transport services and the fob value of the commodity imports represent the carriage insurance and freight (cif) paid value of each imported commodity. But the source regions of the trade and transport services are NOT



identified, and similarly the values of exports of trade and transport services by a region do NOT identify the destination regions. To overcome this lack of information an artificial region called Globe is included in the database. This region collects together all the exports of trade and transport services by other regions, as its imports, and then exports these to other regions to satisfy their demand for the use of trade and transport services associated with commodity imports. By construction the value of imports by Globe for each and every trade and transport margin service must exactly equal the value of exports for the corresponding trade and transport service. However, this does not mean that the trade balance between Globe and each and every region must exactly balance, rather it requires that the sum of Globe's trade balances with other regions is exactly equal to zero. Imbalances between Globe and a particular region are recorded in the capital account for that region, indicating borrowing or lending associated with trade and transport margins.

The R23 database contains a mix of bilateral transactions and with a partial coverage of some transactions where full bilateral data are not available. The remittance data are fully bilateral; each transaction records source and destination regions and values (in US dollars). The data used in the R23 database treats remittances as payments from the household in one region to the household in the destination region. Similarly, bilateral aid transfers by the Donor Assistance Committee members to recipient regions are bilateral. But, recipient regions also receive aid transfers from multilateral/international institutions for which the ultimate source region is unidentified. To overcome this lack of information the Globe region is used as a 'clearing' house: multilateral aid is paid by Globe to the recipient region and the aid budget for Globe is financed by transfers to Globe form other regions. Payments for factor services used in one country/regions but used in another region are also bilateral, i.e., source and destination regions are identified.

An important feature of the construction of a SAM can be deduced from the nature of the entries in the commodity account columns. By definition the column and row totals must equate: these transaction totals can be expressed as an implicit price times a quantity, and the quantity of a commodity supplied must be identical to the quantity of a commodity demanded. The column entries represent the expenditures incurred in order to supply a

Otherwise the global SAM would fail the requirement to be complete and consistent.

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commodity to the economy and hence the implicit price must be exactly equal to the average cost incurred to supply a commodity. Moreover, since the row and column totals equate and the quantity represented by each corresponding entry must be same for the row and column total the implicit price for the row total must be identical to average cost incurred to supply the commodity. Hence the column entries identify the components that enter into the formation of the implicit prices in the rows, and therefore identify the price formation process for each price in the system. Typically, a SAM is defined such that the commodities in the rows are homogenous and that all agents purchase a commodity at the same price.

Total income to the activity account is identified by the row entries. In the simple representation of production in the database each activity makes a single commodity and each commodity is made by a single activity. The expenditures on inputs used in production are recorded in the activity columns. Activities use intermediate inputs, which in this version of the database are record as composites of domestically produced and imported commodities, primary inputs and pay taxes on production and factor use. For each region the sum of the payments to primary inputs and on production and factor use taxes by the activity equals the region's value added measure of GDP.

The remaining accounts relate to the institutions in the SAM. All factor incomes are distributed to the single private household after making allowance for depreciation of physical capital, the payment of direct (income) taxes on factor incomes and payments for foreign owned factors. Incomes from factor sales, government transfers, domestic and foreign, and remittances are the sources of income to the household account. Four categories of expenditures by the household account are recorded; direct (income) taxes, savings, remittances and consumption. The government receives incomes from commodity taxes, production taxes and direct taxes on factor and household incomes, and aid transfers; and uses that income to pay for consumption, aid transfers and for savings. This contrasts with the GTAP database which uses a reduced form representation of intra institutional transactions (see McDonald and Thierfelder, 2004b). There are therefore five sources of savings in each

Thus, in this representation, the activity 'pays' gross operating surplus (GOS) for the factor capital.

McDonald and Sonmez (2004) demonstrate that this limitation of the database can be overcome by adding limited amounts of information **and** by making the strong assumption that the implicit capital account balances are accurate; this assumption is problematic given that the GTAP database appears to estimate the implicit capital account balances as residuals. The R23 database therefore represents one attempt at enhancing the intra-institutional and capital account balances.



region: depreciation, household/private savings, government savings, balances on trade in margin services and balances on the 'current' account, but only a single expenditure activity – investment (commodity) demand<sup>10</sup>.

As should be apparent from the description of the SAM for a representative region the database is strong on inter-regional transactions but relatively parsimonious on intra-regional transactions.

Investment demand therefore includes stock changes, which is arguably problematic since it breaches the implicit presumption of a stable equilibrium value/volume of investment.

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# Overview of the Model

#### 3.1 A Basic 123 Model<sup>11</sup>

The basic 123 model has **one** country with **two** production sectors and **three** commodities. A key objective of the model is to demonstrate the incorporation of the Armington trade insight into a CGE model in the simplest possible context, and demonstrate the consistency of the model with (neoclassical) trade theory. In its simplest form the model contains no factor markets, government or investment, and can be expressed in 14 equations.

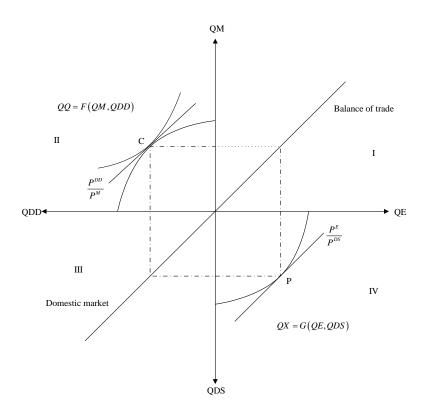
The basic mechanics of the model can be illustrated in a four-quadrant trade diagram (see Deverajan, *et. al.*, 1990) (Figure 3.1), where all the dimensions are positive. The bottom right-hand quadrant of this diagram illustrates how, in this model, domestic production, represented by the production possibility frontier (ppf), can either be consumed domestically, commodity QDS, or exported, commodity QE. The commodities QDS and QE are presumed to be imperfect substitutes; hence the ppf is concave to the origin. Given the relative prices of QDS and QE, i.e.,  $Pe/P_{ds}$ , represented by the budget constraint, the producer decides the optimal (relative) quantities of QDS and QE to produce so as to maximise 'profit', i.e., at the point P. If the exchange rate is the numéraire, and it is assumed (for the moment) that the balance of trade is zero, and the export,  $P_e$ , and import,  $P_m$ , prices are equal, then the relationship between exports and imports is one to one, and the balance of trade condition, in quadrant I, is represented by a 45° line through the origin. Similarly, the transformation of domestic commodity production, QDS, into the domestic commodity for consumption, QDD, is a one to one mapping, and hence the domestic market is characterised by a 45° line through the origin in quadrant III

The origins of the 123 model appear to lie in the 1980s in two papers by de Melo and Robinson (1985 and 1989). The most comprehensive statement of the 123 model is, probably, Deverajan *et al.*, (1990).

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Figure 3.1 A Basic 1\*2\*3 Model



The information in quadrants III, IV, and I can now be used to map out the consumption possibility frontier (cpf) in the remaining quadrant, II. Well-being/utility is maximised where the line representing the relative prices of the domestic commodity and the imported commodity  $P_{dd}/P_m$ , is tangential to the cpf, C, which is also where the indifference curve is at a tangent to the cpf. In this case where world prices are equal and trade is balanced, it is clear that the cpf is a mirror image of the ppf; this is a special case chosen for simplicity of exposition.

The data requirements can be presented as a Social Accounting Matrix (SAM), see Table 3.1. Note how in this SAM all income is spent on consumption in the current period and how there is balanced commodity trade. The former assumption cannot be relaxed because of the absence of a capital account, but the latter assumption can be relaxed by allowing the exogenously determined current account of the balance of payments to be in surplus or deficit via a transfer to the household from the rest of the world. It is relatively simple to extend the 123 model to include factor, government and saving-investment accounts (see Deverajan *et al.*, 1997; and McDonald, 2006).



Table 3.1 Social Accounting Matrix for a Basic 1\*2\*3 Model

|               | Commodity | Activity | Household | Rest of<br>World | Total |
|---------------|-----------|----------|-----------|------------------|-------|
| Commodity     | 0         | 0        | 100       | 0                | 100   |
| Activity      | 80        | 0        | 0         | 20               | 100   |
| Household     | 0         | 100      | 0         | 0                | 100   |
| Rest of World | 20        | 0        | 0         | 0                | 20    |
| Total         | 100       | 100      | 100       | 20               | 0     |

## Armington 'Insight'

Standard neoclassical models presume that all commodities are tradable and all tradable commodities are perfect substitutes. This is scarcely plausible, since such models have a propensity to yield extreme specialisation and wild swings in relative prices when world prices or trade policies change. Moreover, it does not accord with observed patterns of trade, even when the data are recorded with very high degrees of disaggregation (several thousand categories of commodities). The Salter (1959) and Swan (1960) models recognised this by distinguishing between 'tradables' and 'nontradables', which resolved the problem at least at a theoretical level, but had little impact on empirical work. In the Salter-Swan model the link between the domestic and world prices depends solely upon whether a commodity/sector is traded. If a commodity is traded the domestic price equals the world price (*cif* or *fob*). If a commodity is non-traded the domestic price is determined by the interaction of demand and supply on the domestic market. Thus tradables and nontradables are treated asymmetrically.

The Armington insight addresses the difficulties with the modified shares approach to modelling trade in input-output based models and extends the Salter-Swan approach by assuming imperfect substitutability; hence all sectors are treated symmetrically. The link between domestic and world prices then depends upon the trade shares and the elasticities of substitution. For any given elasticity the domestic price will be closer to the world price the greater the trade share, and similarly for any given trade share the domestic price will be closer to the world price the greater the elasticity (see de Melo and Robinson, 1985).



This ensures there are links between domestic and world prices, as in the standard neoclassical model, but these are less strong than in the standard model. Also the model can accommodate two-way trade within the same sector, intra-industry trade; a feature of trade flows that is commonly observed. The approach is therefore consistent with a view of trade, imports and exports, as taking place in differentiated products and is consistent with the Salter-Swan model in that it gives rise to normally shaped offer curves. The exchange rate is also appropriately defined: if the domestic commodity is the numéraire, i.e.,  $P^d$  is set equal to one, then the exchange rate, R, is the real exchange rate of neoclassical theory (the relative price of tradables (QM & QE) to nontradables (QDD & QDS)). If R is set equal to one then  $P^d$  defines the real exchange rate, while for other choices of numéraire R is a monotonic transformation of the real exchange rate.

Typically, despite their limitations, it is common practice to use constant elasticity of substitution (CES) and constant elasticity of transformation (CET) functions for consumption (CES) and production (CET). Specifically, a general (primal) form, *QX*, of the CES/CET specification can be written as

$$QX = \overline{A} \left[ \alpha . QDS^{\rho} + (1 - \alpha) . QE^{\rho} \right]^{1/\rho}$$
(1)

and the price dual, P, as

$$P = \overline{A}^{-1} \left[ \alpha^{1/(1-\rho)} . PDS^{\rho/(\rho-1)} + (1-\alpha)^{1/(1-\rho)} . PE^{\rho/(\rho-1)} \right]^{\rho-1/\rho}$$
(2)

where the substitution/transformation elasticity is given by

$$\sigma \operatorname{or} \Omega = \frac{1}{(1-\rho)} \qquad -\infty < \rho < +1 \tag{3}$$

The export supply and import demand functions are then given by

$$\frac{QE}{QDS} = \left[\frac{(1-\delta).PE}{\delta.PDS}\right]^{\Omega} \tag{4}$$

$$\frac{QM}{QD} = \left[\frac{\beta.PD}{(1-\beta).PM}\right]^{\sigma} \tag{5}$$

where  $\beta$  and  $\delta$  are the respective share parameters and  $\sigma$  and  $\Omega$  are the elasticities.



Exploiting Euler's theorem for linearly homogeneous functions the dual price equations for export transformation and import aggregation can be written as expenditure identities, i.e.,

$$PQ = \frac{PM.QM + PD.QD}{QD} \tag{6}$$

$$PX = \frac{PE.QE + PD.QD}{QX} \tag{7}$$

This approach to the modelling of trade is adopted for the R23 model. The key differences between the R23 model and the Basic 123 model are that world prices are endogenous and each region imports from and exports to multiple other regions in the R23 model. In the basic 123 model, each country is small – world prices are exogenous and each country trades with a single partner, the rest of the world.

# 3.2 Behavioural Relationships in the R23 Model

The within regional behavioural relationships in the R23 model (the behavioural relationships are summarised in Table 3.2 and the variables are summarised in Table 3.3) are typical; given the degree of aggregation there is relatively little room to make them more elaborate. The focus here is therefore on international trade relationships. The activities are assumed to maximise profits using technology characterised by Constant Elasticity of Substitution (CES) and/or Leontief production functions between aggregate primary inputs and aggregate intermediate inputs, with CES production functions over primary inputs and Leontief technology across intermediate inputs. The household maximises utility subject to the price of the consumption good after meeting its direct tax obligations and saving.

The Armington assumption is used for trade. Domestic output is distributed between the domestic market and exports according to a two-stage Constant Elasticity of Transformation (CET) function. In the first stage the domestic producer allocates output to the domestic or export market according to the relative prices for the commodity on the domestic market and the composite export commodity, where the composite export commodity is a CET aggregate of the exports to different regions – the distribution of the exports between regions being determined by the relative export prices to those regions. Consequently, domestic producers are responsive to prices in the different markets – the domestic market and all other regions in



the model – and adjust their volumes of sales in each market according to relative prices. The elasticities of transformation are region specific.

Domestic demand is satisfied by a composite commodity that is formed from domestic production sold domestically and composite imports. This process is modelled by a two-stage CES function. At the bottom stage the composite import commodity is a CES aggregate of imports from different regions with the quantities imported from different regions being responsive to relative prices. The top stage defines a composite consumption commodity as a CES aggregate of a domestic commodity and a composite import commodity with the mix being determined by the relative prices. The elasticities of substitution are level and region specific. Hence, the optimal ratios of imports to domestic commodities and exports to domestic commodities are determined by first order conditions based on relative prices. The price and quantity systems are described in greater detail below

Most commodity and activity taxes are expressed as *ad valorem* tax rates, <sup>12</sup> while income taxes are defined as fixed proportions of household incomes. Import duties and export taxes apply to imports and exports, while sales taxes are applied to all domestic absorption, i.e., imports are subject to sequential import duties and sales taxes, and VAT is applied to household demand. Production taxes are levied on the value of output, while activities also pay taxes on the use of specific factors. Factor income taxes are charged on factor incomes after allowance for depreciation after which the residual income is distributed to households. Income taxes are taken out of household income and then the households are assumed to save a proportion of disposable income. This proportion is either fixed or variable according to the closure rule chosen for the capital account.

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The GLOBE and STAGE models contain code for other methods of levying taxes, e.g., quantity based rates. These codes can be easily adapted for the R23 model.

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 Table 3.2
 Behavioural Relationships for the R23 Model

|                  | Commodities   | Activities                                    | Factors                                     | Households                  | Government                       | Capital                    | Margins  | Rest of<br>World                  | Prices                         |
|------------------|---|---|---|-----------------------------|----------------------------------|----------------------------|--|-----------------------------------|--------------------------------|
| Commodities      | 0   | Leontief Input-<br>Output<br>Coefficients     | 0   | Utility Function            | Fixed Exogenously                | Fixed Shares of<br>Savings | Two-Stage CET<br>Functions                       | Two-Stage CET<br>Functions        | Consumer<br>Commodity<br>Price |
| Activities       | Total Supply from Domestic Production                 | 0   | 0   | 0                           | 0                                | 0                          | 0  | 0                                 | Activity<br>Prices             |
| Factors          | 0   | Two-stage CES Production Functions            | 0   | 0                           | 0                                | 0                          | 0  | Shares of factor income to Globe  | Factor<br>Prices               |
| Households       | 0   | 0   | Fixed Shares of Factor Income               | 0                           | 0                                | 0                          | 0  | Shares of remittances from source |                                |
| Government       | Ad valorem tax<br>rates<br>Specific Tax rates         | Ad valorem tax rates on Output and Factor Use | Average tax rates                           | Average tax rates           | 0                                | 0                          | 0  | Shares of aid from source         |                                |
| Capital          | 0   | 0   | Shares of Factor<br>Incomes                 | Shares of household income  | Government<br>Savings (Residual) | 0                          | Current Account<br>'Deficit' on<br>Margins Trade | Current Acscount 'Deficit'        |                                |
| Margins          | Fixed Technical<br>Coefficients                       | 0   | 0   | 0                           | 0                                | 0                          | 0  | 0                                 |                                |
| Rest of<br>World | Two-Stage CES<br>Functions                            | 0   | Shares of factor incomes after depreciation | Shares of after tax incomes | Shares of GDP                    | 0                          | 0  | 0                                 |                                |
| Prices           | Producer Prices Domestic and World Prices for Imports | Value Added<br>Prices                         | •   |                             |                                  |                            |  |                                   |                                |





Government expenditure consists of commodity (final) demand, which is assumed to be fixed in real/volume terms. Hence government saving, or the internal balance, is defined as a residual. However, the closure rules for the government account allow for various permutations. In the base case it is assumed that the tax rates and volume of government demand are fixed and government savings are calculated as a residual. However, the tax rates can all be endogenously adjusted by making them variable thereby producing an estimate of the constrained optimal tax rate. If the analyst wishes to apply different but fixed tax rates for different instruments this can be achieved by leaving the tax instruments as parameters but changing their values. Equally the volume of government consumption can be changed by adjusting the closure rule. The patterns of government expenditure are altered by changing the parameters that controls the pattern of government expenditure (*qgdconst*).

Total savings come from the households, the internal balance on the government account and the external balance on the trade account. The external balance is defined as the difference between the value of total inflows from the rest of the world (total exports, foreign factor income received, workers' compensation received, remittances received and aid received) and outflows to the rest of the world (total imports, foreign factor income paid, worker's compensation paid, remittances paid and aid paid) converted into domestic currency units using the exchange rate. In the base model it is assumed that the exchange rates are flexible and hence that the external balances are fixed. Alternatively, the exchange rates can be fixed and the external balances can be allowed to vary. Expenditures by the capital account consist solely of commodity demand for investment. In the base solution it is assumed that the shares of investment in total domestic final demand are fixed and that household savings rates adjust so that total expenditures on investment are equal to total savings, i.e., the closure rule presumes that savings are determined by the level of investment expenditures. The volumes of investment can be changed exogenously or solved for endogenously.



Table 3.3 Transactions Relationships for the R23 Model

|               | Commodities  | Activities  | Factors   | Households  |
|---------------|--|---|---|---|
| Commodities   | 0  | (PQD*QINTD)   | 0   | (PQD*(1+TV)*QCD)  |
| Activities    | (PDS*QDS)  | 0   | 0   | 0   |
| Factors       | 0  | $\left( W\!F_{_{\! f}} * F\!D_{_{\! f}} \right)$  | 0   | 0   |
| Households    | 0  | 0   | $\sum_{f} \binom{hvash_f}{*YF_f}$   | 0   |
| Government    | $ \begin{pmatrix} TM_{w} * PWM_{w} \\ *QMR_{w} * ER \end{pmatrix} $ $ \begin{pmatrix} TE_{w} * PWE_{w} \\ *QER_{w} * ER \end{pmatrix} $ $ (TS * PQS * QQ) $ $ (TV * PQD * QCD) $ | $egin{aligned} egin{pmatrix} TX*PX*QX \ TF_f*WF_f \ *WFDIST_f*FD_f \ \end{pmatrix} \end{aligned}$ | $\begin{pmatrix} \textit{TYF}_f * \begin{pmatrix} \textit{YF}_f - \\ \textit{deprec}_f \\ *\textit{YF}_f \end{pmatrix} \end{pmatrix}$ | $(\mathit{TYH}*\mathit{YH})$                              |
| Capital       | 0  | 0   | $\left(\mathit{deprec}_f * \mathit{YF}_f \right)$   | $\binom{YH}{*(1-TYH)}$ * $(SHH)$                          |
| Margins       | $(PT*QT_{_{\scriptscriptstyle{W}}})$   | 0   | 0   | 0   |
| Rest of World | $egin{pmatrix} PWMFOB_{_{W}} \ *QMR_{_{W}}*ER \end{pmatrix}$   | 0   | $\left(\mathit{YFOUT}_{\scriptscriptstyle w,f}\right)$  | $\left(\textit{REMITOUT}_{\scriptscriptstyle w,h}\right)$ |
| Total         | (PQD*QQ)   | (PX * QX)   | $Y\!F_f$  | YH  |

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 Table 3.3 (cont)
 Transactions Relationships for the R23 Model

|               | Government                                      | Capital     | Margins  | RoW  |
|---------------|---|-------------|--|--|
| Commodities   | (PQD*QGD)                                       | (PQD*QINVD) | $\begin{pmatrix} PWE_{_{W}}*QER_{_{W}} \\ *ER \end{pmatrix}$ | $\begin{pmatrix} PWE_{_{W}}*QER_{_{W}} \\ *ER \end{pmatrix}$ |
| Activities    | 0   | 0           | 0  | 0  |
| Factors       | 0   | 0           | 0  | $\left(\mathit{YFIN}_{f,w}\right)$                           |
| Households    | 0   | 0           | 0  | $\left(\textit{REMITIN}_{h,w}\right)$                        |
| Government    | 0   | 0           | 0  | $(AID\_B\_IN_{g,w})+$<br>$(AID\_M\_IN_{g,w})$                |
| Capital       | (YG-EG)   | 0           | $(\mathit{KAPREG}*\mathit{ER})$                              | $(\mathit{KAPREG}*\mathit{ER})$                              |
| Margins       | 0   | 0           | 0  | 0  |
| Rest of World | $(AID\_B\_OUT_{w,g})+$<br>$(AID\_M\_OUT_{w,g})$ | 0           | 0  | 0  |
| Total         | YG  | INVEST      | 0  | 0  |



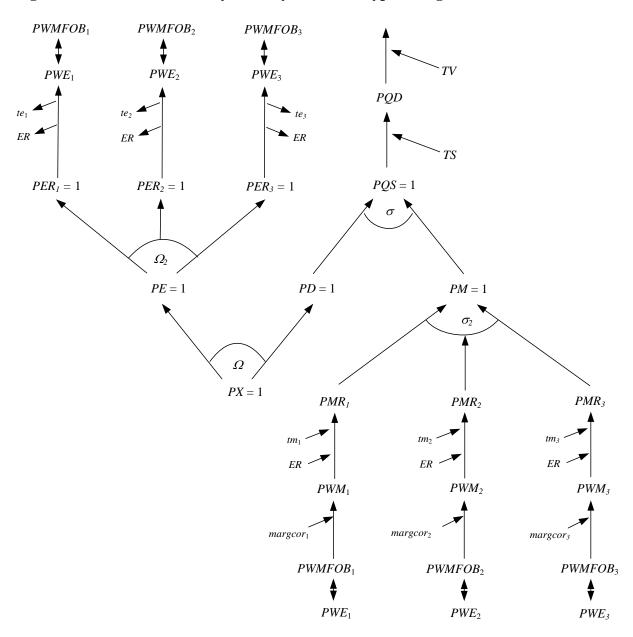
#### 3.2.1 Price System

The price system is built up using the principle that the components of the 'price definitions' for each region are the entries in the columns of the SAM. Hence there are a series of explicit accounting identities that define the relationships between the prices and thereby determine the processes used to calibrate the tax rates for the base solution. However, the model is set up using a series of linear homogeneous relationships and hence is only defined in terms of relative prices. Consequently, as part of the calibration process it is necessary to set some of the prices equal to one (or any other number that suits the modeller) – this model adopts the convention that prices are normalised on the basic prices, i.e., at the level of the CES and CET aggregator functions: *PQS*, the supply price of the domestic composite consumption commodity, and *PXC*, the producer price of the composite domestic output. The price system for a typical region in a 4-region global model is illustrated by Figure 3.2 – note that this representation abstracts from the Globe region.

The relationships between the various prices in the model are illustrated in Figure 3.2. The basic prices of the commodities supplied (PQS) are determined by the domestic prices of the domestically supplied commodities (PD) and the domestic prices of the composite imports (PM). Then the domestic consumer prices (PQD) are defined by the addition of sales taxes (TS) that are levied on all domestic demand, and the prices paid by households are further increased by the addition of VAT (TV). The prices of the composite imports are determined as aggregates of the domestic prices paid for imports from all those regions that supply imports to this economy (PMR) under the maintained assumption that imports are differentiated by their source region. The region specific import prices are expressed in terms of the domestic currency units after paying for trade and transport services and any import duties. Thus a destination region is assumed to purchase a commodity in a source economy where the price is defined in "world dollars" at the basket exchange rate and is valued free on board (fob), i.e., *PWMFOB*. The carriage insurance and freight (cif) price (PWM) is then defined as the fob price plus trade and transport margin services (margcor) times the unit price of margin services (PT). The cif prices are related to the domestic price of imports by the addition of any import duties (TM) and then converted into domestic currency units using the nominal exchange rate (ER).



Figure 3.2 Commodity Price System for a Typical Region



The prices for output (PX) are determined by the domestic prices (PD) and the composite export prices (PE). The composite export prices are a CET aggregates of the export prices received by the source economy for exports to specific destinations (PER). The prices of the composite exports are determined as aggregates of the domestic prices paid for exports by all those regions that demand exports from this economy under the maintained assumption that exports are differentiated by their destination region. The prices paid by the destination regions (PWE) are net of export taxes (TE) and are expressed in the currency units of the model's reference region by use of the nominal exchange rate. Note how the export prices by



region of destination (*PER*), and the intermediate aggregates, are all normalised on 1, but the seeming counterpart of normalised import prices by source region (*PMR*) are not normalised on 1. The link between the regions is therefore embedded in the identification of the quantities exchanged rather than the normalised prices and is a natural consequence of the normalisation process. The CET function can be switched off so that the domestic and export commodities are assumed to be perfect substitutes; this is the assumption in the GTAP family and related families of models, and is an option in this model.

The price system also contains a series of equilibrium identities. Namely the fob export price (PWE) for region x on its exports to region y must be identical to the fob import price (PWMFOB) paid by region y on its imports from region x. These equilibrium identities are indicated by double headed arrows.

# 3.2.2 Quantity System

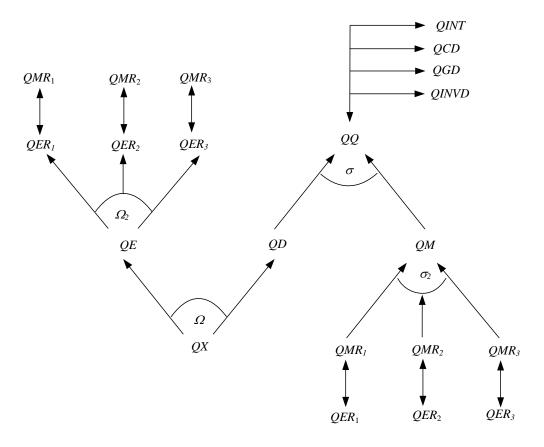
The quantity system for a representative region is somewhat simpler (Figure 3.3). The composite consumption commodity (QQ) is a mix of the domestically produced commodity (QD) and the composite import commodity (QM), where the domestic and imported commodities are imperfect substitutes. The equilibrium conditions require that the quantities imported from different regions (QMR) are identical to the quantities exported by other regions to the representative region (QER); the equilibrium identities are indicated by double headed arrows.

The composite consumption commodity is then allocated between domestic intermediate demands (QINT), private consumption demand (QCD), government demand (QGD) and investment demand (QINVD).

On the output side, domestic output (QX) is allocated between the domestic market (QD) and composite export commodities (QE) under the maintained assumption of imperfect transformation. Exports are allocated between the different destination regions (QER) under the maintain assumption of imperfect transformation.



Figure 3.3 Quantity System for a Typical Region



# 3.2.3 Production System

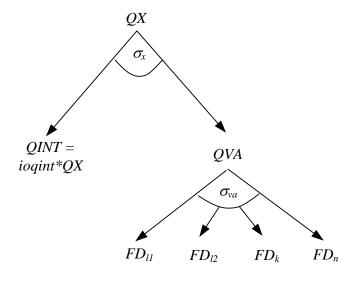
The production system is set up as a two-level nest of CES production functions (Figures 3.4 and 3.5). At the top level the intermediate input commodity (QINT) is combined with aggregate primary inputs (QVA) to produce the output of an activity (QX). This top level production function can take either CES or Leontief form, with CES being the default and the elasticities being activity **and** region specific.<sup>13</sup> The value added production function is a standard CES function over capital, land and two types of labour, with the elasticities being region specific. The notation for all primary inputs, natural and aggregates, is the same: quantity demand is  $FD_f$ , quantity supplied is  $FS_f$  and the factor price is  $WF_f$ . (where f refers to the particular factor). The operation of this aggregator function can, of course, be influenced by choices over the closure rules for the factor accounts.

The model allows the user to specify the share of intermediate input cost in total cost below which the Leontief alternative is automatically selected. The user also has the option to make activity and region specific decisions about the selection of CES or Leontief forms.

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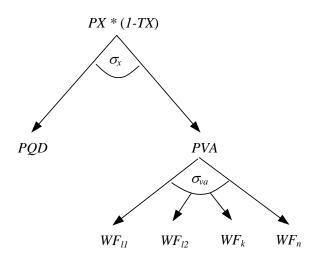
Figure 3.4 Production Quantity System for a Typical Region



The model defines all factors as being potentially defined as being characterised by un/underemployment. The default setting is full employment for all factors and it is left to the user to decide whether any or all factors in any or all countries/regions experience un/underemployment in the data with which the model is calibrated.

In the price system for production the value added prices (PVA) are determined by the activity prices (PX), the production tax rates (TX), and the price of the intermediate input (PQD). The price of value added is determined by the factor prices ( $WF_f$ ) and any factor use taxes ( $tf_f$ ).

Figure 3.5 Production Price System for a Typical Region





# 3.3 The Globe Region

An important feature of the model is the use of the concept of a region known as Globe. While the R23 database contains complete bilateral information relating to the trade in commodities, i.e., in all cases transactions are identified according to their region of origin and their region of destination, this is not the case for trade in margins services associated with the transportation of commodities. Rather the R23 database identifies the demand, in value terms, for margin services associated with imports by all regions from all other regions but does not identify the region that supplies the margin services associated with any specific transaction. Consequently, the data for the demand side for margin services is relatively detailed but the supply side is not. Indeed, the only supply side information is the total value of exports of margin services by each region. The Globe construct allows the model to get around this shortage of information, while simultaneously providing a general method for dealing with any other transactions data where full bilateral information is missing. The Globe region is used in this model to account for payments and incomes from factors used in one region but owned in another region where the source and destination regions are not simultaneously identified and for the disbursement of multilateral aid, which is funded by regions, but distributed by multilateral agencies.

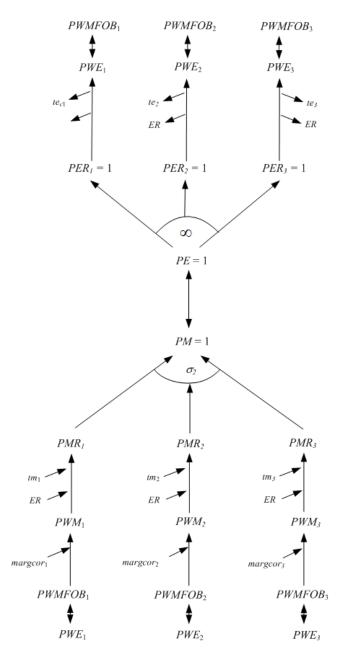
The price system for the Globe region is illustrated in Figure 3.6. On the import side Globe operates like all other regions. The commodities used in trade and transport services are assumed to be differentiated by source region and the proportion of imports accounted for by the source region. Thus a CES aggregation is used. It is assumed that imports of trade and transport services can *potentially* incur trade and transport margins (*margcor*) and face tariffs (*TM*); in fact, the database does not include any transport margins or tariff data for margin services in relation to the destination region, although they can, and do, incur export taxes levied by the exporting region.

The export side is slightly different. In effect the Globe region is operating as a method for pooling differentiated commodities used in trade and transport services and the only differences in the use of trade and transport services associated with any specific import are the quantities of each type of trade service used and the mix of types of trade services. Underlying this is the implicit assumption that each type of trade service used is homogenous, and should be sold, therefore, at the same price. Hence, the export price system for Globe



needs to be arranged so that Globe exports at a single price, i.e., there should be an infinite elasticity of substitution between each type of trade service exported irrespective of its destination region. Therefore, the average export price (PE) should equal the price paid by each destination region (PER), which should equal the export price in world currency units (PWE) and will be common across all destinations (PT).

Figure 3.6 Price System for the Globe Region

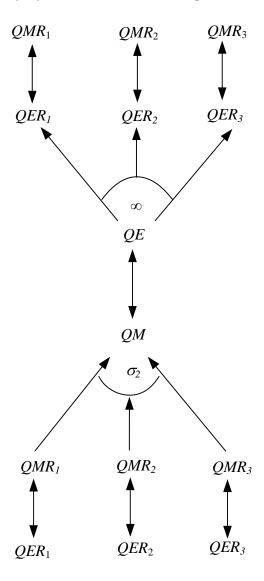


The linked quantity system contains the same asymmetry in the treatment of imports and exports by Globe (see Figure 3.7). The imports of trade and transport commodities are



assumed to be differentiated by region and the proportion of imports accounted for by the source region, hence the elasticity of substitution is greater than or equal to zero but less than infinity, while the exports of trade and transport commodities are assumed to be homogenous and hence the elasticities of transformation are infinite.

Figure 3.7 Quantity System for the Globe Region



One consequence of using a Globe region for trade and transport services is that Globe runs trade balances with all other regions. These trade balances relate to the differences in the values of trade and transport commodities imported from Globe and the value of trade and transport commodities exported to Globe; however, the sum of Globe's trade balances with other regions must be zero since Globe is an artificial construct rather than a real region. But the demand for trade and transport services by any region is determined by technology, i.e.,





the coefficients, *margcor*, and the volume of imports demanded by the destination region. This means that the prices of trade and transport commodities only have an indirect effect upon their demand – the only place these prices enter into the import decision as a variable is as a partial determinant of the difference between the *fob* and *cif* valuations of other imported commodities. Consequently, the primary market clearing mechanism for the Globe region comes through the quantity of trade and transport commodities it chooses to import.

The Globe concept has other potential uses in the model. All transactions between regions for which there is an absence of full bilateral information can be routed through the Globe region. While this is not a 'first best' solution, it does provide a 'second best' method by which augmented versions of the GTAP database can be used to enrich the analyses of international trade in a global model prior to availability of full bilateral transactions data (see McDonald and Sonmez (2006) for a first application).



# **Formal Description of the Model**

This formal description of the model proceeds in five stages with three of them in this section and the fourth, relating to model closure rules, being detailed in the next section. The fifth stage is in Appendix A2 where all the equations are summarised. For this section the description begins with identification of the sets used in the model, this is followed by details of each equation (block) in the model and ends with a table that summarises all the equations and identifies the associated variables, the counts for equations and variables and identifies whether the equation is implemented or not for the Globe region.

#### 4.1 Model Sets

Rather than writing out each and every equation in detail it is useful to start by defining a series of sets; thereafter if a behavioural relationship applies to all members of a set an equation only needs to be specified once. The natural choice for this model is a set for all the transactions by each region (*sac*) plus a series of sets that group commodities, activities, factors, import duties, export taxes, trade margins, trade and finally some individual accounts relating to domestic institutions.<sup>14</sup> The outer set for any region is defined as

$$sac = \{c, a, ff, h, tmr, ter, tff, g, i, marg, w, total\}$$

and the following are the basic subsets for each region in this model

-

In the R23 model, there is one sector so the set c has a single entry 'ccom' and the set a has a single entry 'acom'. In the equation listings that follow, we drop the c and a indices.

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```
c(sac) = \{commodities\}
  a(sac) = \{activities\}
 ff(sac) = \{ \text{natural and aggregate factors} \}
   f(ff) = \{\text{natural factors}\}
   h(sac) = \{\text{household}\}\
tmr(sac) = \{ad \ valorem \ import \ duties\}
 ter(sac) = \{export taxes\}
 tff(sac) = \{factor use taxes\}
  g(sac) = \{saltax, vattax, prodtax, facttax, dirtax, govt, dac, grant\}
    gt(g) = \{govt\}
   i(sac) = \{kap\}
  m(sac) = \{ trade and transport margins \}
  w(sac) = \{ rest \text{ of the world - trade partners and aggregates} \}
      It is also necessary to define a set of regions, r, for which there are two subsets
       rgn(r) = \{ all regions excluding Globe \}
       ref(r) = \{reference regions for global numeraire\}
and a subset of w is needed to allow for Globe
       wgn(w) = \{\text{Rest of world without Globe}\}\
```

Various subsets of r are declared and then assigned on the basis of certain characteristics of the database used to calibrate the specific implementation of the model, so-called dynamic sets. The subsets of r used in the model are

```
rqx(r) = \{\text{Regions with CES function at Level 1 of nest}\}\

rqxn(r) = \{\text{Regions without CES function at Level 1 of nest}\}\
```



```
rt(r) = \{\text{regions with margin exports}\}\
rtn(r) = \{\text{regions without margin exports}\}\
re(r) = \{\text{regions with exports}\}\
ren(r) = \{\text{regions without exports}\}\
rer(r, w) = \{\text{region pairs with exports}\}\
rern(r, w) = \{\text{region pairs without exports}\}\
rm(r) = \{\text{regions with imports}\}\
rm(r) = \{\text{regions without imports}\}\
rmr(w, r) = \{\text{region pairs with imports}\}\
rmr(w, r) = \{\text{region pairs without imports}\}\
rx(r) = \{\text{regions with domestic production}\}\
rxn(r) = \{\text{regions without domestic production}\}\
rd(r) = \{\text{regions producing and demanding domestically}\}\
rdn(r) = \{\text{regions not producing and demanding domestically}\}\
```

A macro SAM that facilitates checking various aspects of model calibration and operation is used in the model and this needs another set of macro accounts, ss,

$$ss = \begin{cases} commdty, activity, valuad, hholds, \\ tmtax, tetax, govtn, kapital, margs, world, totals \end{cases}$$

The model also makes use of a series of mapping files that are used to link sets. 15 These are

-

Given the SAM structure for the database, the mapping sets are needed to link taxes to the commodity against which the taxes are paid – for example import tariffs on imports from region *x* must apply to imports from region *x*.

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```
map\_w\_tmr(w,tmr) = \{Ad\ valorem\ tariff\ mapping\}
map\_tmr\_w(tmr,w) = \{Ad\ valorem\ tariff\ mapping\ reverse\}
map\_w\_ter(w,ter) = \{Export\ tax\ mapping\}
map\_ter\_w(ter,w) = \{Export\ tax\ mapping\ reverse\}
map\_w\_marg(m,w) = \{Trade\ margin\ mapping\ of\ marg\ to\ w\}
map\_marg\_w(m,w) = \{Trade\ margin\ mapping\ of\ w\ to\ marg\}
map\_r\_w(r,w) = \{Region\ to\ trade\ partner\ mapping\}
map\_w\_r(w,r) = \{Region\ to\ trade\ partner\ mapping\}
map\_f\_tff(f,tff) = \{Factor\ taxes\ to\ factors\}
map\_tff\_f(tff,ff) = \{Factor\ taxes\ to\ factors\ reverse\}
```

Finally, various other sets are declared to facilitate model operation. These are

$$sacn(sac) = \{SAM \text{ accounts without totals}\}\$$
  
 $ssn(ss) = \{Macro SAM \text{ accounts without totals}\}\$ 

## 4.1.1 Reserved Names

The model uses a number of names that are reserved; these are

IMPTAX Import Duties
EXPTAX Export Taxes
SALTAX Sales Taxes
VATTAX Value Added Taxes
PRODTAX Production Taxes
DIRTAX Direct Taxes

The model also has a reserved name for the Globe region ('glob') and for the Globe region as a trade partner ('wglob').

## 4.1.2 Conventions

The equations for the model are set out in 11 'blocks' each of which can contain a number of sub blocks. The equations are grouped under the following headings:

#### 1. TRADE BLOCK

- a. Exports Block
- b. Imports Block

#### 2. COMMODITY PRICE BLOCK



- 3. NUMERAIRE PRICE BLOCK
- 4. PRODUCTION BLOCK
  - a. Production
- 5. FACTOR BLOCK
- 6. HOUSEHOLD BLOCK
  - a. Household Income
  - b. Household Expenditure
- 7. GOVERNMENT BLOCK
  - a. Government Tax Revenues
  - b. Government Income
  - c. Government Expenditure Block
- 8. KAPITAL BLOCK
  - a. Savings Block
  - b. Investment Block
- 9. MARKET CLEARING BLOCK
  - a. Factor Accounts
  - b. Commodity Accounts
  - c. Commodity Trade Accounts
  - d. Margin Trade Accounts

#### 10. MACROECONOMIC CLOSURE BLOCK

- a. Trade balances
- b. Absorption Closure
- 11. MACROECONOMIC AGGREGATES BLOCK
- 12. SLACKS BLOCK

This grouping is carried throughout the model code, i.e., it is followed for the parameter declaration and calibration, variable declaration and variable initialization sections. This modularization of the code is adopted for ease of reading and altering the model rather than being a requirement of the model.

A series of conventions are adopted for the naming of variables and parameters. These conventions are not a requirement of the modelling language; rather they are designed to ease reading of the model.



- All VARIABLES are in upper case.
- The standard prefixes for variable names are: P for price variables, Q for quantity variables, W for factor prices, F for factor quantities, E for expenditure variables, Y for income variables, and Y for value variables
- All variables have a matching parameter that identifies the value of the variable in the base period. These parameters are in upper case and carry a '0' suffix, and are used to initialise variables.
- All parameters are in lower case, except those paired to variables that are used to initialise variables.
- Parameter names have a two or five character suffix which distinguishes their definition, e.g., \*\*sh is a share parameter, \*\*av is an average and \*\*const is a constant parameter.
- For the Armington (CES) functions all the share parameters are declared with the form *delta*\*\*, all the shift/efficiency parameters are declared with the form *ac*\*\*, and all the elasticity parameters are declared with the form *rho*\*\*, where \*\* identifies the function in which the parameter operates.
- For the CET functions all the share parameters are declared with the form *gamma*\*\*, all the shift/efficiency parameters are declared with the form *at*\*\*, and all the elasticity parameters are declared with the form *rho*\*\*, where \*\* identifies the function in which the parameter operates.
- All coefficients in the model are declared with the form *io*\*\*\*\*, where \*\*\*\* consists of two parts that identify the two variables related by the coefficient.
- The index ordering follows the specification in the SAM: row, column, and then r to indicate the region. For example, exports from region r to region w would be  $QER_{c,w,r}$  because region r's export data in its SAM is found in the commodity row and the trade partner column (w). Likewise, imports in region r from region w are designated,  $QMR_{w,c,r}$  because region r's import data in its SAM is found in the trade partner row (w) and the commodity column.
- All sets have another name, or alias, given by the set name followed by "p". For example, the set of commodities may be called f or fp.



#### **4.2** Equations for the Model

The model equations are reported and described by blocks/groups below and then they are summarised in table A2 in the appendix.

## 4.2.1 Exports Block Equations

The treatment of exports is complicated by the incorporation of the facility to treat export commodities as imperfect substitutes and by the need to accommodate the special case of exports (of trade and transport services) that are homogenous from Globe. The presumption of imperfect substitution is the default presumption in this model; reasons for this decision being its symmetry with the Armington assumption on the imports side, the amelioration of the terms of trade effects associated with the Armington assumption and a belief that, in general, there is differentiation between commodities supplied to domestic and export markets. However, there are proponents of the arguments for treating exports as perfect substitutes and there are clearly cases where such an assumption may be appropriate, e.g., supplies of unprocessed mineral products.<sup>16</sup>

The domestic prices of commodity exports by destination, w, and source, r, region (PER) are defined as the product of world prices of exports (PWE) – also defined by destination and source region, the source region's exchange rate (ER) and one minus the export tax rate  $(TE)^{17}$  (X1). Also note that the concept of a single world price for a commodity is not a valid concept since, by definition, world prices are endogenous and therefore ALL regions are treated as 'large' producers of a commodity. Hence, an extreme version of the small country assumption is not supported but the shares of total exports of a commodity accounted for by each region will be a determinant of each region's impact on the average world price.

The GTAP model assumes perfect substitution and historically it has been argued that perfect substitution is appropriate for Australia. It has been argued (Dixon, *et. al.*, 1992) that the use of CET functions for exports can be avoided by increasing the degree of disaggregation, however, this is not appropriate for the 123 and R23 models and arguably unviable for any (practical) global model. This argument also lacks symmetry since the same argument could be adopted with respect to imports and the Armington insight. There are, however reasons, to be cautious about the use of CET functions (see McDonald, 2011).

In the R23 model ALL taxes are expressed as having positive values; any 'tax' rate with a negative value is a 'subsidy' rate.

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## **Export Block Equations**

$$PER_{w,r} = PWE_{w,r} * ER_r * (1 - TE_{w,r}) \qquad \forall r \in rer$$
(X1)

$$PE_r * QE_r = \sum_{w} PER_{w,r} * QER_{w,r} \qquad \forall r \in rgn$$
(X2)

$$PER_{w,r} = PE_r \quad \forall r \notin rgn, w \in wgn$$
(X3)

$$QX_{r} = at_{r} \cdot \left(\gamma_{r} * QE_{r}^{\rho_{r}^{l}} + (1 - \gamma_{r}) * QD_{r}^{\rho_{r}^{l}}\right)^{\frac{1}{l}\rho_{r}^{l}}$$

$$\forall r \in (rd \cap re), r \in rgn$$
(X4)

$$QE_{r} = QD_{r} \left[ \frac{PE_{r}}{PD_{r}} * \frac{(1 - \gamma_{r})}{\gamma_{r}} \right]^{\frac{1}{(\rho_{r}' - 1)}} \qquad \forall r \in (rd \cap re), r \in rgn$$
(X5)

$$QX_{r} = QD_{r} + QE_{r} \qquad \forall (r \in (rd \cap ren), r \in rgn)$$

$$OR \ \forall (r \in (rdn \cap re), r \in rgn)$$
(X6)

$$QER_{w,r} = QE_r * \left(\frac{PER_{w,r}}{\left(\left(PE_r * \gamma_{w,r}^r * at_r^r\right)^{\rho_r^e}\right)}\right)^{\left(\frac{1}{\rho_r^e - 1}\right)} \quad \forall r \in rgn \text{ and } r \in rer$$
(X7)

$$QE_r = \sum_{w} QER_{w,r} \qquad \forall r \notin rgn$$
(X8)

$$QE_r = QM_r \qquad \forall r \notin rgn \tag{X9}$$

The prices of the composite export commodities can then be expressed as simple volume weighted averages of the export prices by region, where  $PE_r$  and  $QE_r$  the price and quantity of the composite export commodity from region r, and the weights are the volume shares of exports and are variable. This comes from the fact that a CET function is linear homogenous and hence Eulers theorem can be applied. Notice however that (X2) is only implemented of the set rgn, i.e., the region Globe is excluded. Rather the composite export price for trade margin commodities from Globe is defined in X3, which indicates that it is assumed that the trade margin commodities exported by Globe are perfect substitutes for each



other, i.e., the same price is paid for each trade margin commodity by ALL purchasing regions.

Domestic commodity outputs (QX) are either exported (QE) as composite commodities or supplied to the domestic market (QD). The allocation of output between the domestic and export markets is determined by the output transformation functions, Constant Elasticity of Transformation (CET) functions, (X4) with the optimum combinations of QE and QD determined by first-order conditions (X5). The model allows for the possibility, however remote, that for some regions commodities are non-traded: in such cases domestically produced commodities are non-traded - by definition the quantity supplied to the domestic market is the amount produced - or that a commodity may be produced domestically and exported but not consumed domestically. These two sets of possibilities are covered by X6.

These quantity equations deal only with the composite export commodities, i.e., hypothetical commodities whose roles in the model are to act as neutral intermediaries that enter into the first-order conditions that determine the optimal mix between domestic use and exports of domestic commodity production (X5). In the model the composite export commodities are themselves CET aggregates of commodity exports to different regions (*QER*), and the appropriate first order condition is given by (X7).

Note that (X7) does not define the exports of trade margin commodities by Globe; this is because these commodities are assumed to be perfect substitutes and therefore simple addition is adequate, this is done by X8. Finally, there is a need for an equilibrium conditions for trade by Globe. Since Globe is an artificial construct whose sole role in the model is to gather exports, whose destinations are unknown, and supply imports, whose sources are unknown, and vice versa, it must always balance its trade within each period. Thus the volume of exports of trade margin commodities by Globe must be exactly equal to the volume of imports of trade margin commodities, see X9.

#### 4.2.2 Imports Block Equations

The prices of imported commodities are made up of several components. The export price in foreign currency units – valued free on board (*fob*) (*PWMFOB*) – plus the cost of trade and transport services, which gives the import price carriage insurance and freight (*cif*) paid (*PWM*), plus any import duties; all of which are then converted into domestic currency units



(*PMR*). Clearly the import price value *fob* (*PWMFOB*) is identical to the export price valued *fob* (*PWE*) – this condition is imposed in the market clearing block (see below) – and hence the *cif* price is defined in M1, where *margcor* is the quantity of trade and transport services required to import a unit of the imported commodity and *PT* is the price of trade and transport services. Embedded in the definition of the coefficient *margcor* is the explicit assumption that transporting a commodity from a specific source to a specific destination requires the use of a specific quantity of services – the actual cost of these services can vary according to changes in the prices of the trade and transport services. But, the quantity of services required to transport a particular commodity is exogenously determined by the 'technical' coefficients (*margcor*); the 'technical' coefficients are parameters in the model.

## Import Block Equations

$$PWM_{w,r} = PWMFOB_{w,r} + \left(margcor_{w,r} * PT_r\right) \qquad \forall r \in rmr$$
(M1)

$$PMR_{w,r} = PWM_{w,r} * ER_r * (1 + TM_{w,r}) \qquad \forall r \in rmr$$
(M2)

$$PM_{r} * QM_{r} = \sum_{w \text{scmr}_{w,r}} \left( PMR_{w,r} * QMR_{w,r} \right) \qquad \forall rm_{r}$$
(M3)

$$QQ_r = ac_r \left( \delta_r * QM_r^{-\rho_r^c} + \left( 1 - \delta_r \right) * QD_r^{-\rho_r^c} \right)^{-\left( \frac{1}{\rho_r^c} \right)} \qquad \forall r \in rgn, r \in rx, r \in rm$$
(M4)

$$QM_{r} = QD_{r} * \left( \left( \frac{PD_{r}}{PM_{r}} \right) * \left( \frac{\delta_{r}}{\left( 1 - \delta_{r} \right)} \right) \right)^{\frac{1}{\left( 1 + \rho_{r}^{c} \right)}} \qquad \forall r \in rgn, r \in rx, r \in rm$$
(M5)

$$QQ_r = QD_r + QM_r \qquad \forall r \in rgn, r \in rx, r \in rm$$
(M6)

$$QMR_{w,r} = QML_r * \left(\frac{PMR_{w,r} * acr_r^{\rho_r^m}}{PM_r * \delta_{w,r}^r}\right)^{\left(-\frac{1}{\rho_r^m + 1}\right)} \forall r \in rmr$$
(M7)

$$QT_{w,r} = QMR_{w,r} * margcor_{w,r} \qquad \forall r \in rgn$$
(M8)



The domestic prices of imports from a region (PMR) are then defined (M2) as the product of world prices of imports (PWM) – after payment for carriage, insurance and freight (cif) - the exchange rate (ER) and one plus the *ad valorem* import tariff rate (TM).

The prices of the composite import commodities can be expressed as a simple volume weighted averages of the of the import prices by region, (M3) where  $PM_r$  and  $QM_r$  are the price and quantity of the composite import commodity by region r, and the weights are the volume shares of imports and are variable. Notice however that (M3) is only controlled by the set cm, in contrast to (X2) – the composite export price – which was also controlled by the set rgn, i.e., the region Globe was excluded. This reflects the fact that the region Globe does import commodities using the same trading assumption as other regions but only exports homogenous trade and transport services, which explains the need for the equation (X3).

The composite (consumption) commodities (QQ) are then a mixture of composite imports (QM) and domestic demand from domestic production (QD). The mixtures between the domestic and import supplies are determined by the substitution functions, Constant Elasticity of Substitution (CES) functions, (M4) with the optimal combinations of QM and QD being determined by first-order conditions, (M5). But, some commodities may be nontraded and therefore M6 and M7 are implemented only if the commodity is traded, which leaves QQ undefined for non-traded commodities. By definition, if there are no imports the quantity demanded by the domestic market is the amount produced, and if there is no domestic production the amount demanded is the amount imported. These two sets of possibilities are both covered by M6.

The composite imports of commodities are defined as CES aggregates of the imports from different regions (QMR) (M7). The first order conditions come from the price definition terms for composite imports and are only implemented for those cases where there were import transactions in the base period – this is controlled by the set rm(r). Initially the formulation of M7 is not intuitive but the formulation is a straightforward manipulation of a more conventional representation.

Also associated with any imported commodity is a specific quantity of trade and transport services. These services are assumed to be required in fixed quantities per unit of import by a specific region from another specific region, (M8) where the *margcor* are the



trade and transport coefficients associated with a unit (quantity) import by region r from region w. This is only implemented for trade and transport commodities and for regions that 'actually' import goods (rgn).

#### 4.2.3 Commodity Price Block Equations

The composite price equation (CP1) is derived from the first order conditions for tangencies to consumption possibility frontiers. By exploiting Euler's theorem for linearly homogeneous functions the composite prices can be expressed as expenditure identities rather than dual price equations for export transformation and import aggregation, such that PQS is the weighted average of the producer price of a commodity, when PD is the basic price of domestically produced commodities and PM the domestic (basic) price of the composite imported commodity, QD the quantity of the domestic commodity demanded by domestic consumers, QM the quantity of composite imports and QQ the quantity of the composite commodity. Note how the commodity quantities are the weights and since PQS is a weighted average of two basic prices it is also a basic price.

This composite commodity price (CP1) does not include sales taxes (TS), which create price wedges between the purchaser price of a commodity (PQD) and the producer (basic) prices (PQS). Hence the purchaser price is defined as the producer price plus the sales taxes (CP2). This formulation means that the sales taxes are levied on all sales on the domestic market, irrespective of the origin of the commodity concerned.

#### Commodity Price Block Equations

$$PQS_{r} = \frac{\left(PD_{r} * QD_{r}\right) + \left(PM_{r} * QM_{r}\right)}{QQ_{r}} \qquad \forall r \in rgn$$
(CP1)

$$PQD_r = PQS_r * (1 + TS_r) \qquad \forall r \in rgn$$
(CP2)

#### 4.2.4 Numéraire Price Block

It is necessary to define a price numéraire for each region; for this model two standard domestic numéraires are coded, which gives the user an option. The producer price index





 $(PPI_r)$  for each region is given by the price received by producers on the domestic market (PD) and therefore does not require a separate equation. However the consumer price indices  $(CPI_r)$  require defining since the model does not include a variable for the final prices paid by consumers: the CPIs are defined as the purchasers price plus the value added tax (N1). An advantage of with using the CPI as the numéraire is that the transactions within a region are expressed in real terms without the need to adjust values for price changes.

#### Numéraire Price Block Equations

$$CPI_r = PQD_r * (1 + TV_r)$$
  $\forall r \in rgn$  (N1)

$$ERPI = \sum_{ref} vqesh_{ref} *ER_{ref}$$
(N2)

The exchange rate numéraire (N2) is defined as an exchange rate index for the set *ref*, which allows the user to define the reference region/regions for the exchange rate. If *ref* is declared as a single member set then the exchange rate of that region is the numéraire. <sup>18</sup> Fixed country trade balances must be seen as specified in "real" terms defined by the global numéraire. So, if the US exchange rate is fixed to one, the global numéraire is defined in US dollars, and all trade balances can be seen as "real" variables defined in terms of the value of US exports. If the weighted exchange rate for a group of regions is chosen as global numéraire, trade balances can be seen as "claims" against a weighted average of exports by the group of regions.

Note how both the consumer and producer price indices are implemented only for those regions that have consumption and production activities. Hence the Globe does not have its own price indices; rather the price indices for Globe are those of the reference region(s) in the model.

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A guiding principle for selecting the membership of *ref* is to choose a region or group of regions that accounts for a large share of global trade and output. The group of regions can be defined as all regions in the model except for the Globe region.

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#### 4.2.5 Production Block Equations

The output price (PX) is defined as the share weighted average of the prices received on the domestic and export markets, where the weights are quantity shares (P1). The value of output by region is defined as the activity price (PX) less production taxes (TX) times the volume of output (QX): this revenue must be divided between payments to primary inputs – the price of value added (PVA) times the quantity of value added (QVA) – and intermediate inputs – the price of intermediate inputs (PQD) times the volume of aggregate intermediate inputs (QINT) (P2).

The default top level production function (P3), is a CES aggregation of primary and intermediate inputs, where the first order conditions for profit maximization (P4) determine the optimal ratio of the inputs. The efficiency factor (ADX) and the factor shares parameters ( $\delta_r^x$ ) are calibrated from the data and the elasticities of substitution, from which the substitution parameters are derived ( $\rho_r^x$ ), are exogenously imposed. Note in this case, the efficiency factor is declared as variable; typically, this is fixed in the closure and market clearing conditions but it is useful to have this as a variable for certain types of simulation exercises. If the user decides to forego the use of CES function for the production function (P3) then the option to assume a Leontief aggregation is available. The Leontief functions require that aggregate intermediate inputs (P5) and aggregate values added (P6) are fixed proportions of the volumes of output.

The second level production functions are CES aggregation functions over the primary factors that are demanded by each activity (FD), with efficiency factors (ADVA) and the factor shares ( $\mathcal{S}_r^{va}$ ) calibrated from the data and the elasticities of substitution, from which the substitution parameters are derived ( $\mathcal{P}_r^{va}$ ), are exogenously imposed (P7). Again the efficiency parameters are defined as variables for convenience when implementing certain types of simulation. The associated first-order conditions for optimal factor combinations are derived from equalities between the wage rates for each factor in each activity and the values of the marginal products of those factors in each activity, (P8). The total factor payments in a region

The code allows the user to select a Leontief at the top level by setting the substitution elasticity equal to zero. This is equivalent to the standard production structure in the GTAP model. The user also has an option to set a minimum (value) share of intermediate inputs below which the Leontief option is selected.

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are the wage rates (WF) times the factor quantities. Note however that the relevant prices in the first-order condition are defined as inclusive of any factor use taxes/subsidies (TF) associated with the use of those factors in that region.

### Production Block Equations - Top Level

$$PX_{r} = \frac{\left(PD_{r} * QD_{r}\right) + \left(PE_{r} * QE_{r}\right)}{QX_{r}} \quad \forall r \in rgn, r \in rx$$
(P1)

$$PX_r * (1 - TX_r) * QX_r = (PVA_r * QVA_r) + (PQD_r * QINT_r) \quad \forall r \in rgn$$
(P2)

$$QX_{r} = ADX_{r} * \left[ \delta_{r}^{x} * \left( QVA_{r} \right)^{-\rho_{r}^{x}} + \left( 1 - \delta_{r}^{x} \right) * \left( QINT_{r} \right)^{-\rho_{r}^{x}} \right]^{-1/\rho_{r}^{x}}$$

$$\forall r \in rgn, r \in rqx$$
(P3)

$$QVA_{r} = QINT_{r} * \left( \left( \frac{PQD_{r}}{PVA_{r}} \right) * \left( \frac{\delta_{r}^{x}}{\left( 1 - \delta_{r}^{x} \right)} \right) \right)^{\left( \frac{1}{\left( 1 + \rho_{r}^{x} \right)} \right)} \forall r \in rgn, r \in rqx$$

$$(P4)$$

$$QINT_r = ioqintqx_r * QX_r \qquad \forall r \in rgn, r \in rqxn$$
 (P5)

$$QVA_r = ioqvaqx_r * QX_r \qquad \forall r \in rgn, r \in rqxn$$
(P6)

$$QVA_{r} = ADVA_{r} * \left( \sum_{f} \delta_{f,r}^{va} * \left( ADFD_{f,r} * FD_{f,r} \right)^{-\rho_{r}^{va}} \right)^{\left(-\frac{1}{\rho_{r}^{va}}\right)} \forall r \in rgn$$

$$(P7)$$

$$\begin{split} WF_{f,r} * \left( 1 + TF_{f,r} \right) &= PVA_r * QVA_r * \left[ \sum_{f} \delta_{f,r}^{va} * \left( ADFD_{f,r} * FD_{f,r} \right)^{-\rho_r^{va}} \right]^{(-1)} \\ * \delta_{f,r}^{va} * \left( ADFD_{f,r} \right)^{-\rho_r^{va}} * \left( FD_{f,r} \right)^{\left(-\rho_r^{va} - 1\right)} & \forall r \in rgn, \delta_{f,r}^{va} > 0 \end{split}$$

(P8)

While production uses intermediate inputs these do not require separate specification since there is only a single commodity.



#### 4.2.6 Factor Block Equations

The total domestic factor income received by each factor account (YF) is defined as the summation of the earnings of that factor across all activities (F1). The payments of factor earning outside of a region (YFOUT) are defined as fixed shares (yfoutsh) of domestic factor incomes after depreciation (deprec), i.e., it is assumed that all depreciation is realised in the region where the factor is employed (F2). These payments are bilateral with the behavioural relationship being defined by the fixed shares (yfoutsh). Factor payments to a region (YFIN) are then simply defined as the mirror of the factor payments out (F3). Note how F3 changes the region source and destination labels on YFOUT to identify factor payments that come in to a region. It is necessary to include allowances for any exchange rate changes so remittances out must be converted to the currency units of the global exchange rate using the source regions exchange rate and then converted into the currency units of the destination region.

#### Factor Block Equations

$$YF_{f,r} = \sum_{a} WF_{f,r} * FD_{f,r} \qquad \forall r \in rgn$$
(F1)

$$YFOUT_{w,f,r} = yfoutsh_{w,f,r} * \left(YF_{f,r} * \left(1 - deprec_{f,r}\right)\right) \qquad \forall r \in rgn, w \in wgn$$
(F2)

$$YFIN_{f,w,r} = \left(\sum_{w\$map\_r\_w;r\$map\_w\_r} \frac{YFOUT_{w,f,r}}{ER_r}\right) * ER_r \qquad \forall r \in rgn, w \in wgn$$
(F3)

$$YFDIST_{f,r} = \begin{pmatrix} \left(YF_{f,r} * \left(1 - deprec_{f,r}\right)\right) \\ + \sum_{w} YFIN_{f,w,r} - \sum_{w} YFOUT_{w,f,r} \end{pmatrix} * \left(1 - TYF_{f,r}\right) \qquad \forall r \in rgn$$
(F4)

The factor incomes available for distribution to the domestic institutional accounts (*YFDIST*) are defined as the proportions of domestic factor incomes after deprecation retained



within each region plus factor incomes from abroad (*YFIN*), after payment of any factor income taxes (*TYF*) (F4).

## 4.2.7 Household Block Equations

Households acquire income from the sale of factor services, the distributed income from the sale, domestically and overseas, of factor services (*YFDIST*) and from government transfers. Household income (*YH*) is defined simply as the sum of factor incomes available for distribution plus transfers from the government (*hogovconst*) (H1). Note the transfer payments are constant but scalable through changes in *HGADJ*, which is fixed to one in the base case. By making the scaling factor a variable the user can, for instance, endogenously determine the change is transfers from the government to 'compensate' households for policy induced changes in household incomes.

#### Household Block Equations

$$YH_{r} = \sum_{f} hvash_{f,r} *YFDIST_{f,r} + \left(HGADJ * hogovconst_{r} * CPI_{r}\right) \quad \forall r \in rgn$$
(H1)

$$REMITOUT_{w,h,r} = remitoutsh_{w,h,r} * (YH_r * (1 - TYH_r))$$
  $\forall r \in rgn, w \in wgn$  (H2)

$$REMITIN_{h,w,r} = \left(\sum_{w \$ map\_r\_w; r \$ map\_w\_r} \frac{REMITOUT_{w,h,r}}{ER_r}\right) * ER_r \quad \forall r \in rgn, w \in wgn$$
(H3)

The resources available to households for consumption are more complex. Households can remit funds to households in other regions (*REMITOUT*), which it is assumed is done in fixed shares (*remitoutsh*) of after direct tax (*TYH*) income (H2); hence it is assumed that all income taxes are paid by the household that receives the distributed incomes from factor sales plus transfer income, i.e., transfer incomes are taxed. Since there are full bilateral data on



remittances, remittance income to a region (*REMITIN*) is simply defined as the mirror of the remittances paid out (H3), with the behavioural relationship being determined by the 'choice' to make remittance payments. Note how H3 changes the region source and destination labels on *REMITOUT* to identify remittance incomes, and includes the necessary allowance for any exchange rate changes: remittances out must be converted to the currency units of the global exchange rate using the source regions exchange rate and then converted into the currency units of the destination region.

Household consumption demand (*QCD*) is then defined by the household income after paying taxes, making and receiving remittances and saving, which is then divided by the unit price, inclusive of value added tax (VAT), of the consumption good (H4). Note how the saving rates are defined as proportions of after tax and remittances incomes that are saved; this is important for the calibration of the income tax and savings parameters.

The prices for final demand are specified as inclusive of *ad valorem* value added taxes (*TV*), which is additional to other 'general' sales taxes (*TS*) levied on domestic demand. This does not allow for differential realised rates of rebate of VAT on intermediate input demand – the implicit presumption is that the rebate rates are equal to 100 percent or that the rebate rate does not change in any simulation. It is recommended that in the R23 model the GST rates (*TS*) are NOT used as a tax replacement instrument and that they are not changed in experiments.<sup>20</sup>

#### 4.2.8 Government Tax Block Equations

There are eight tax instruments that are all defined as simple *ad valorem* rates dependent upon the values of imports, exports, sales, household demand, production and factor use by activities and the levels of factor and household and income. All tax rates are variables in the R23 model. Typically, these tax rates are fixed, but by declaring the tax rates as variables it is possible to design simulations for tax replacement exercises that define optimal tax rates.

The government tax revenue equations simply sum the revenues from each different tax instrument for each region. These are not strictly necessary for the model since the equations

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De facto this advice means that sales taxes rates should be treated as an unchanging distortion in the underlying data and simulations are conducted under the implicit assumption that the degree of distortion is unchanged.

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could be collapsed into a single government income equation (see G1 below). However, it is useful to carry around these additional variables since they can then be used in model closures to accommodate specific government tax revenue objectives and they are useful sources of information when analysing simulation results.

#### Government Tax Revenues Equations

$$MTAX_{r} = \sum_{w} \left( TM_{w,r} * PWM_{w,r} * ER_{r} * QMR_{w,r} \right) \qquad \forall r \in rgn, r \in rm$$
(T1)

$$ETAX_{r} = \sum_{w} \left( TE_{w,r} * PWE_{w,r} * ER_{r} * QER_{w,r} \right) \qquad \forall r \in rgn, r \in re$$
(T2)

$$STAX_r = \left(TS_r * PQS_r * \left(QINT_r + QCD_r + QGD_r + QINVD_r\right)\right)$$

$$\forall r \in rgn, (r \in re \text{ or } rm)$$
 (T3)

$$VTAX_r = (TV_r * PQD_r * QCD_r)$$
  $\forall r \in rgn, (r \in re \text{ or } rm)$  (T4)

$$ITAX_{r} = (TX_{r} * PX_{r} * QX_{r})$$
 
$$\forall r \in rgn$$
 (T5)

$$FYTAX_{r} = \sum_{f} \left( TYF_{f,r} * \left( YF_{f,r} - \left( deprec_{f,r} * YF_{f,r} \right) \right) \right) \qquad \forall r \in rgn$$
 (T6)

$$HTAX_{r} = (TYH_{r} * YH_{r})$$
  $\forall r \in rgn$  (T7)

$$FTAX_{r} = \sum_{f} \left( TF_{f,r} * WF_{f,r} * FD_{f,r} \right) \qquad \forall r \in rgn$$
(T8)

Ad valorem import duty revenues (MTAX) are defined as total  $ad\ valorem$  import duty revenue in region r (T1) from import duty rates (TM); Export tax revenues (ETAX) are defined as total export tax revenue in region r (T2) from export tax rates (TE); sales tax revenues (STAX) are defined as total sales tax revenue in region r (T3) from sales tax rates (TS); VAT revenues (TAX) are defined as total VAT revenue in region T (T4) from VAT rates (TX); production tax revenues (TAX) are defined as total production tax revenue in region T (TS) from production tax rates (TX); factor income tax revenues (TXX) are defined as total factor income tax revenue in region T (TS) from factor income tax revenue in region T (TS) from tax revenues (TXX) are defined as total household income tax revenue in region T



(T7) from income tax rates (TYH), and factor use tax revenues (FTAX) are defined as total factor use tax revenue in region r (T8) from factor use tax rates (TF).

#### 4.2.9 Government Block Equations

Government income (YG) is defined as the sum of government tax revenues (G1), where the tax revenues are treated as expenditures by the accounts paying the taxes and hence are defined in the tax revenues block. The government also gets income from both bilateral and multilateral aid payments. The equation is defined over all regions. The region Globe does not collect tax revenue; YG for Globe is the total multilateral aid.

#### **Other Government Equations**

$$YG_{r} = MTAX_{r} + ETAX_{r} + STAX_{r} + VTAX_{r} + ITAX_{r}$$

$$+FYTAX_{r} + HTAX_{r} + FTAX_{r}$$

$$+\sum_{g,w} AID_{B_{i}} B_{i} IN_{g,w,r} + \sum_{g,w} AID_{M_{i}} IN_{g,w,r} \qquad \forall r$$
(G1)

$$EG_r = PQD_r * QGD_r + (HGADJ_r * hogovconstr * CPI_r) \qquad \forall r \in rgn$$
 (G2)

$$AID\_B\_OUT_{w,g,r} = aid\_b\_outsh_{w,g,r} * GDP_r \qquad \forall r \in rgn, w \in wgn, g \in gt$$
 (G3)

$$AID\_B\_IN_{g,w,r} = \left(\sum_{w\$map\_r\_w,r\$map\_w\_r} \frac{AID\_B\_OUT_{w,g,r}}{ER_r}\right) * ER_r$$

$$\forall r \in rgn, w \in wgn, g \in gt$$
(G4)

$$AID\_M\_OUT_{w,g,r} = aid\_m\_outsh_{w,g,r} * GDP_r \quad \forall r \in rgn, g \in gt$$
 (G5a)

$$AID\_M\_OUT_{w,g,r} = aid\_m\_outsh_{w,g,r} * YG_r \quad \forall r \notin rgn, g \in gt$$
 (G5b)

$$AID\_M\_IN_{g,w,r} = \left(\sum_{w\$map\_r\_w;r\$map\_w\_r} \frac{AID\_M\_OUT_{w,g,r}}{ER_r}\right) * ER_r \quad \forall g \in gt$$
(G6)

Government expenditures include purchases of commodities and transfers to households (G2). Government demand for commodities is assumed fixed in real terms, i.e., the volume is fixed, but can be adjusted: the precise specification depends upon the choice of closure rule



(see below). Note that government expenditures are defined without aid transfers, i.e., they are defined as DOMESTIC expenditures.

Bilateral aid transfers ( $AID\_B\_OUT$ ) are defined as fixed shares ( $aid\_b\_outsh$ ) of GDP (G3). Since the outward aid transfers are bilateral, the aid transfers from region r to region w are identical to the transfer received by region r from region w after adjustments for the exchange rates. Hence, the aid inflows ( $AID\_B\_IN$ ) are equal to the matched outflows ( $AID\_B\_OUT$ ) converted to the currency units of the global exchange rate using the source regions exchange rate and then converted into the currency units of the destination region (G4).

Multilateral aid transfers (*AID\_M\_OUT*) are defined as fixed shares of *GDP* for all non-globe regions (G5a). Since the Globe region is a clearing house, and therefore does not have *GDP*, aid transfers from Globe are defined as fixed shares of government income *YG*. For Globe, government income only comes from bilateral or multilateral aid flows and there is no tax revenue (G1). Multilateral aid payments received in a region (*AID\_M\_IN*) are payments from globe. Hence the aid inflows (*AID\_M\_IN*) are equal to the matched outflows (*AID\_M\_OUT*) converted to the currency units of the global exchange rate using the source regions exchange rate and then converted into the currency units of the destination region (G6).

#### 4.2.10 Capital Account Block Equations

Income to the capital (savings and investment) account, total savings (*TOTSAV*), comes from household savings, depreciation allowances, government savings (*KAPGOV*) and the surplus on the capital account of the balance of payments (*KAPWOR*), and hence is simply defined as the sum of savings by domestic and 'foreign' agents (K1). In this model the household savings rates are declared as variables (*SHH*) that define the proportions of income saved after the payment of income taxes; again this simply increases the flexibility available for simulations. Note that household savings are taken out of after-tax income.



#### Savings-Investment Block Equations

$$TOTSAV_{r} = \left( \left( (YH_{r} * (1 - TYH_{r})) - \sum_{w,h} REMITOUT_{w,h,r} + \sum_{h,w} REMITIN_{h,w,r} \right) * (SHH_{r}) \right)$$

$$+ \sum_{f} (deprec_{f,r} * YF_{f,r})$$

$$+ KAPGOV_{r} + (KAPWOR_{r} * ER_{r}) \qquad \forall r \in rgn$$

$$INVEST_{r} = (PQD_{r} * QINVD_{r}) \qquad \forall r \in rgn$$
(K1)

Government savings are calculated as residual (see the *KAPGOV* equations, MC3.1, below). The surplus on the capital account (*KAPWOR*) is defined in terms of the foreign currency (see MC3.4 and MC3.3) and therefore the exchange rate appears in this equation (this is a matter of preference).

Investment demand (*QINVD*) is then defined as the value of investable funds (*INVEST*) divided by the unit price of investment goods (*PQD*). This arrangement allows adjustment of investment demand either through the volume, or the expenditure or the value share of final demand (see MC6.2 below).

#### 4.2.11 Market Clearing Block Equations

In the base solution to the model the market clearing, or equilibrium, conditions are relatively straightforward. Factor supplies must equal factor demands plus any unemployed labour in a region (MC1.1). Thus the factor supplies cannot be simply equated to the sum of factor demands in the base period but must equal the sum of factor demand plus any unemployed factors. As such the default specification of the model is NOT full employment but rather that there is the possibility of unemployment for all factors.

The (composite) commodity supplies must equal (composite) commodity demands (MC2.1). It appears that there is no equilibrium condition for the supply of domestic output to the domestic market, which is, in fact, achieved through the commodity output equation that could have been treated as a market clearing equation.



#### Factor Account Equation

$$FS_{f,r} = FD_{f,r} + UNEMP_{f,r} \qquad \forall r \in rgn$$
(MC1.1)

$$UNEMP_{f,r} \ge 0 \qquad \forall r \in rgn \tag{MC1.2}$$

The commodity trade accounts define the market clearing conditions for bilateral trade. The *fob* prices for imports (*PWMFOB*) for all imports by destination and source must be equal to the *fob* prices for exports (*PWE*) by source and destination (MC3.1). In addition the quantities of imports (*QMR*) for all imports by destination and source must be equal to the quantities of exports (*QER*) by source and destination (MC3.2). These equations are not completely straightforward since it is necessary in their implementation to employ mappings between exporting and importing regions that require the 'switching' of labels on accounts within the equation. Globe is a clearing house and each type of transaction – the sale of margin services or multilateral aid flows – must balance globally. The total supply of margin services must equal the total demand for margin services (MC3.3). In a balanced model, the *GLOBESLACK* variable should equal zero. Likewise, total multilateral aid payments made by donor countries must equal the total multilateral aid received in recipient countries (MC3.4). In a balanced model, the *GLOBSLACK3* variable should equal zero.

#### Commodity Account Equations

$$QQ_r = QINT_r + QCD_r + QGD_r + QINVD_r \qquad \forall r \in rgn$$
(MC2.1)

The trade consistency equations do not however deal with the requirements for market clearing with respect to the trade transactions undertaken by the Globe region. However similar conditions apply for the margins trade. These require that the total demand for each and every trade and transport service (QT) is exactly equal to the total exports of that service by Globe (QER) and the import prices of margin services (PT) must equal the export prices (fob) from Globe; these conditions are imposed by (MC4.1) and (MC4.2) respectively.



(MC3.4)

R23 Model

#### Commodity Trade Account Equations

$$PWMFOB_{w,r} = PWE_{w,r} \tag{MC3.1}$$

$$QMR_{w,r} = QER_{w,r} \tag{MC3.2}$$

$$\sum_{w} \left( PWM_{w,r} * QMR_{w,r} \right) = \sum_{w} \left( PWE_{w,r} * QER_{w,r} \right) + GLOBESLACK \quad \forall r \notin rgn \quad (MC3.3)$$

$$\sum_{w} AID \_M \_OUT_{w,g,r} = \left(\sum_{w} AID \_M \_IN_{g,w,r}\right) + GLOBESLACK3 \qquad \forall r \notin rgn$$

#### Margin Trade Account Equations

$$\sum_{w} QT_{w,r} = \sum_{w} QER_{w,"glob"} \qquad \forall r \in rgn$$
(MC4.1)

$$PT_r = PWE_{w,"glob"} \qquad \forall r \in rgn$$
(MC4.2)

#### 4.2.12 Macroeconomic Closure Block Equations

The government account is cleared by defining government savings (KAPGOV) as the difference between government income and government expenditure on consumption, transfers and bilateral and multilateral aid transfers; hence government savings are explicitly treated as a residual, (MC5.1). The deficit/surplus on the current account is computed in two-stages. First, the bilateral trade balances (KAPREG) are calculated as the difference in the value of all revenue sent from region r to region r (import sales from region r valued in world prices, factor income paid to region r value of all revenue region r receives from region r (export sales to region r valued in world prices, factor income received from region r valued in world prices, factor income received from region r valued in world prices, factor income received from region r valued in world prices, factor income received from region r remittances received from region r and bilateral and multilateral aid payments received). First, the bilateral trade balances are calculated for trade flows between regions r and regions other than Globe (MC5.2). The bilateral trade balance between each region r and Globe is r McDonald & Thierfelder



calculated in MC5.3. Each region's bilateral trade balance with Globe includes margin purchases and multilateral aid payments. Note, margin services are purchased from Globe to support imports from region w,  $QT_{w,r}$  – it is not a purchase from region w. These transactions are valued in terms of the global numéraire. Then, the overall balance of trade (KAPWOR) is computed for each region, (MC5.4).

## Investment and Savings Account Equations

$$KAPGOV_{r} = YG_{r} - EG_{r}$$

$$-\sum_{w,g} AID_{B}_{OUT_{w,g,r}} - \sum_{w,g} AID_{M}_{OUT_{w,g,r}} \quad \forall r \in rgn \quad (MC5.1)$$

$$KAPREG_{w,r} = \begin{pmatrix} (PWMFOB_{w,r} * QMR_{w,r}) \\ + \sum_{f} \frac{YFOUT_{w,f,r}}{ER_{r}} + \sum_{h} \frac{REMITOUT_{w,h,r}}{ER_{r}} \end{pmatrix}$$

$$KAPREG_{w,r} = \left\{ + \sum_{f} \frac{1}{ER_{r}} + \sum_{h} \frac{1}{ER_{r}} + \sum_{h} \frac{1}{ER_{r}} + \sum_{g} \frac{AID_{m} - OUT_{w,g,r}}{ER_{r}} + \sum_{g} \frac{AID_{m} - M_{m} - OUT_{w,g,r}}{ER_{r}} \right\}$$

$$- \left( \left( PWE_{w,r} * QER_{w,r} \right) + \sum_{f} \frac{YFIN_{f,w,r}}{ER_{r}} + \sum_{h} \frac{REMITIN_{h,w,r}}{ER_{r}} + \sum_{g} \frac{AID_{m} - M_{m} - IN_{g,w,r}}{ER_{r}} \right)$$

$$+ \sum_{g} \frac{AID_{m} - B_{m} - IN_{g,w,r}}{ER_{r}} + \sum_{g} \frac{AID_{m} - M_{m} - IN_{g,w,r}}{ER_{r}}$$

$$+ \sum_{g} \frac{AID_{m} - B_{m} - IN_{g,w,r}}{ER_{r}} + \sum_{g} \frac{AID_{m} - M_{m} - IN_{g,w,r}}{ER_{r}}$$

(MC5.2)

$$\begin{aligned} \mathit{KAPREG}_{\scriptscriptstyle{w,r}} = & \left( \left( \sum_{\scriptscriptstyle{w}} \mathit{PT}_r * \mathit{QT}_{\scriptscriptstyle{w,r}} \right) + \sum_{\scriptscriptstyle{f}} \frac{\mathit{YFOUT}_{\scriptscriptstyle{w,f,r}}}{\mathit{ER}_r} + \sum_{\scriptscriptstyle{g}} \frac{\mathit{AID} \_\mathit{M} \_\mathit{OUT}_{\scriptscriptstyle{w,g,r}}}{\mathit{ER}_r} \right) \\ & - \left( \left( \mathit{PWE}_{\scriptscriptstyle{w,r}} * \mathit{QER}_{\scriptscriptstyle{w,r}} \right) + \sum_{\scriptscriptstyle{f}} \frac{\mathit{YFIN}_{\scriptscriptstyle{f,w,r}}}{\mathit{ER}_r} + \sum_{\scriptscriptstyle{g}} \frac{\mathit{AID} \_\mathit{M} \_\mathit{IN}_{\scriptscriptstyle{g,w,r}}}{\mathit{ER}_r} \right) \\ & \forall \mathit{w} \not\in \mathit{wgn} \end{aligned}$$

(MC5.3)

$$KAPWOR_r = \sum_{w} KAPREG_{w,r}$$
  $\forall r$  (MC5.4)



The absorption closure equations exist to support closure rule choices relating to the distribution of final demand between the institutions within regions. The value of final demand by region (*VFDOMD*) is simply the sum of the value of final demand across the three sources of final demand – households, government and investment (MC6.1). The value share of final demand accounted for by investment (*INVESTSH*) is defined in (MC6.2), and the value share of final demand accounted for by government (*VGDSH*) is defined in (MC6.3). No similar expression exists for the households because household expenditures are defined after taxes and savings and it is difficult to justify setting the household shares relative to the value of total final demand since this would nullify the distributional consequences of a policy shock. If the expenditure shares by investment and government are fixed then the expenditure share of ALL households will by definition be fixed.

### Absorption Closure Equations

$$VFDOMD_{r} = \left(PQD_{r} * \left(\left(QCD_{r} * (1+TV_{r})\right) + QGD_{r} + QINVD_{r}\right)\right) \qquad \forall r \in rgn$$

$$INVESTSH_{r} * VFDOMD_{r} = INVEST_{r} \qquad \forall r \in rgn$$

$$(MC6.1)$$

$$VGDSH_r * VFDOMD_r = (PQD_r * QGD_r)$$
  $\forall r \in rgn$  (MC6.3)

#### 4.2.13 Macroeconomic Aggregates Block Equations

Strictly the model does not require the specification of any macroeconomic aggregates since such variables in the context of a CGE model are summary measures that do not play an active role in any of the behavioural relationships, i.e., they could all be computed post simulation as summary measures. However, in the context of a dynamic model it may be useful to endogenously compute various macroeconomic variables to facilitate the calibration of the dynamic baseline. The only obvious such variable required, which does not already



exist, e.g., total savings/investment, internal and external balances, etc., is GDP<sup>21</sup>. This is defined, from the expenditure side, in MC7.1.

## Macroeconomic Aggregates Equations

$$GDP_{r} = \left(PQD_{r} * (1+TV)_{r} * QCD_{r}\right) + \left(PQD_{r} * (QGD_{r} + QINVD_{r})\right) + \left(PE_{r} * QE_{r}\right) - \left(PM_{r} * QM_{r}\right) \qquad \forall r \in rgn$$
(MC7.1)

### 4.2.14 Slacks Block Equations

The final equations are for slacks. The satisfaction of Walras's law requires that if all other accounts balance then so must the final account; thus the difference between savings and investment for all regions that save should always be zero and hence the variable *WALRAS*<sub>r</sub> should equal zero in (MC8.1). That this variable equals zero is a good check on the correct specification of the model. Finally, the global trade balance must by definition be zero and hence so must *KAPWORSYS* (MC8.2)

#### Slack Equations

$$TOTSAV_r = INVEST_r + WALRAS_r$$
  $\forall r \in rgn$  (MC8.1)  
 $KAPWORSYS = \sum_r KAPWOR_r$   $\forall r \in rgn$  (MC8.2)

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Other summary measures of total economic activity are specified in the System of National Accounts, e.g., GNI, NDP, etc. These summary measures could be used in preference to GDP.

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## Model Closure Conditions or Rules<sup>22</sup>

In mathematical programming terms the model closure conditions are, at their simplest, a matter of ensuring that the numbers of equations and variables are consistent. However, the economic theoretic dimensions of model closure rules are more complex, and, as would be expected in the context of an economic model, more important. The essence of model closure rules is that they define important and fundamental differences in perceptions of how an economic system operates (see Sen, 1963; Pyatt, 1987; Kilkenny and Robinson, 1990). The closure rules can be perceived as operating on two levels; on a general level whereby the closure rules relate to macroeconomic considerations, e.g., is investment expenditure determined by the volume of savings or exogenously, and on a specific level where the closure rules are used to capture particular features of an economic system, e.g., the degree of intersectoral capital mobility.

This model allows for a range of both general and specific closure rules. The discussion below provides details of some of the options available with this formulation of the model by reference to the accounts to which the rules refer. However, as will become readily apparent there are many permutations available, and hence this discussion deals with the general principles rather than trying to define all possible permutations. The philosophy adopted in the implementation of this model is to define a (minimal) base closure for the replication of the base case and then impose closure changes within a loop in the experiment/simulation file. This approach allows substantial flexibility in the definition of policy simulations while simultaneously allowing sensitivity testing of the chosen model closures. For the base solution the choice of model closure is basically a neoclassical closure; the reason for choosing this closure is NOT a statement about its appropriateness, rather it is a closure that allows flexibility in model solution and hence simplifies the process of verifying that the model is correctly specified and calibrated.

When fixing variables in a closure condition, the variable is fixed at its base value, which is noted as the variable name followed by "0".

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The term model 'closure' is used generically here to cover all permutations including variations in market clearing conditions and macroeconomic closure conditions.

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## **5.1** Foreign Exchange Account Closure

For the world numéraire the exchange rate index for the reference regions (*EPRI*) is fixed (C1a), although a parameter, *numerchk*, is attached to allow for ease of checking the homogeneity of the model.<sup>23</sup> At the same time the exchange rate for Globe is fixed as equal to the world numéraire (C1b).

The default closure for the non-Globe regions involves an assumption that the nominal exchange rate is flexible. Hence the trade balances ( $KAPWOR_{rgn}$ ) for all non-Globe regions are fixed (C1c) and the exchange rate is flexible. The most straightforward alternative is to fix the exchange rates for all non-Globe regions (C1d) and unfix the trade balances. However, there are clearly a range of permutations whereby the exchange rates for some regions are flexible while for others they are fixed.<sup>24</sup> Note, for the non-Globe regions, either equation C1c or equation C1d is used.

#### Foreign Exchange Account Closure

$$EPRI = \overline{EPRI0} * numerchk$$
 (C1a)

$$ER_{"glob"} = \overline{EPRI0} * numerchk$$
 (C1b)

$$KAPWOR_{rgn} = \overline{KAPWOR0_{rgn}} * numerchk$$
 (C1c)

$$ER_{rgn} = \overline{ER0}_{rgn} \tag{C1d}$$

## 5.2 Capital Account Closure

This discussion of the closure rules for the capital account begins by abstracting from some of the complexities into interactions between different closure conditions. To ensure that aggregate savings equal aggregate investment, the determinants of either savings or

In the base and model simulations, numerchk equals one. It is a parameter attached to the world numéraire and the regional numéraire (see C5a and C5b). To check for homogeneity, a value other than one should have no effect on real variables in the model.

The option is provided in the programme to control choice of these, and several other, closures by selection of regions to assign to a set and its complement.

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(C2d)

investment must be fixed. The default closure for this account requires the assumption that savings rates are exogenously determined, which involves fixing the savings rates (*SHH*) (C2a) – typically at their base rates. With such a savings-driven closure the value and volume of investment must be free to vary.

Choosing an investment-driven closure requires making two distinct choices; first, is the level investment to be determined in terms of volume, value or expenditure share of final demand, and second, how will savings adjust. Clearly, again, it is possible that the choices will vary by region. If the investment volume (*QINVD*) is fixed (C2b), then the value of investment expenditure might change due to changes in the prices of investment commodities (*PQD*). If the value of investment (*INVEST*) is fixed (C2c) then both the value of savings must be free to adjust as must the actual volume of investment. The same applies if the shares of investment expenditures in final demand (*INVESTSH*) are fixed (C2d).

#### Capital Account Closure

$$SHH_r = \overline{SHH0}_r$$

$$QINVD_r = \overline{QINVD0}_r$$

$$INVEST_r = \overline{INVEST0}_r$$

$$(C2a)$$

$$INVEST_r = \overline{INVESTSH0}_r$$

$$(C2c)$$

Fixing savings, and thus deeming the economy to be savings-driven, could be termed a Neo-Classical approach. Closing the economy by fixing investment however makes the model reflect a Keynesian investment-driven assumption for the operation of an economy.

However, there are potentially important interaction effects. Note that there are other sources of potential savings for a region – the government and the trade balances. The magnitudes of these other savings sources can also be controlled through the closure rules (see below). Consequently, there will clearly be an important interdependence between the choices of closure rules for different accounts; the most obvious one to be aware of is the





interaction between household savings rates and household income tax rates when tax rates are made flexible and the level of government savings/deficit is fixed.

#### **5.3** Government Account Closure

The closure rules for the government account are slightly more complex because they are important components of the model that are used to investigate fiscal policy considerations. The base specification uses the assumption that government savings are a residual; when the determinants of government income, i.e., tax rates, remittances etc., and expenditure are 'fixed', government savings must be free to adjust.

Thus in the base specification all the tax rates are fixed (C3.1a – C3.1h). Consequently, any changes in tax revenue to the government are consequences of changes in the other variables that enter into the tax income equations (T1 to T8). Controlling the volumes of commodity demand (QGD) in the base specification government expenditure is then achieved by fixing the volumes of commodity demand (QGD) (C3.2a). Notice however that since prices are typically free to change it is necessary to ensure that both government expenditure (EG) in (C3.2b) and the share of government expenditure in final demand (VGDSH) in (C3.2c) must be flexible.

This specification ensures that all the parameters that the government can/does control are fixed and consequently that the only determinants of government income and expenditure that are free to vary are those that the government does not <u>directly</u> control. Hence the equilibrating condition is that government savings, the internal balance ( $KAPGOV_r$ ), is not fixed in (C3.2d).

The number of possible permutations for closing the government account for each region is consequently substantial. Practical experience indicates that great care is needed when adjusting the government closure rules to avoid both unbalancing the model and imposing closure rules that are contradictory. A common change is to require that the government's savings/deficit is fixed; this requires fixing *KAPGOV* in C3.2d, in which case either government income or expenditure must be free to adjust. Such a condition might

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The most common problems with contradictory government closure rules relate to the interactions between the government and capital (investment) accounts, in particular when a combination of flexible savings and tax rates producing mutually contradictory effects.

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reasonably be expected in many circumstances, e.g., the government might define an acceptable level of borrowing or such a condition might be imposed externally.

#### Tax Rate and Revenue Closure

| $TM_r = \overline{TM0}_r$   | (C3.1a) |
|-----------------------------|---------|
| $TE_r = \overline{TE0_r}$   | (C3.1b) |
| $TS_r = \overline{TSO_r}$   | (C3.1c) |
| $TV_r = \overline{TV0_r}$   | (C3.1d) |
| $TX_r = \overline{TX  0_r}$ | (C3.1e) |
| $TYF_r = \overline{TYF0_r}$ | (C3.1f) |
| $TYH_r = \overline{TYH0_r}$ | (C3.1g) |
| $TF_r = \overline{TF0_r}$   | (C3.1h) |
|                             |         |

In its simplest form this can be achieved by allowing one of the previously fixed tax rates (C3.1a to C3.1h) to vary. Thus if the value added tax rate (TV) is made variable then the VAT rates will vary so as to satisfy the internal balance condition. More complex experiments might result from the imposition of multiple conditions, e.g., a halving of import duty rates coupled with a reduction in government deficit, in which case the variables TM and KAPGOV would also require resetting. But these conditions might create a model that is infeasible, e.g., due to insufficient flexibility through the import duties mechanism, or unrealistically high rates of VAT. In such circumstances it may be necessary to allow adjustments in multiple tax adjusters.

Note also that as with the investment account, there is a needed care over setting the constraints on government demand. If the government demand volume (QGD) is fixed (C3.2a), then the value of government expenditure might change due to changes in the prices of commodities (PQD). If the value of government expenditure (EG) is fixed (C3.2b) then both government savings must be free to adjust as must the actual volume of expenditure,



when tax rates are exogenous. The same applies if the shares of government expenditures in final demand (*VGDSH*) are fixed (C3.2e).

## Government Expenditure and Savings Closure

$$QGD_r = \overline{QGDO_r}$$
 (C3.2a)

$$EG_r = \overline{EGO_r}$$
 (C3.2b)

$$VGDSH_r = \overline{VGDSH0_r}$$
 (C3.2c)

$$KAPGOV_r = \overline{KAPGOVO_r} * numerchk$$
 (C3.2d)

## 5.4 Technology and Efficiency

The default assumption in the model is that efficiency, commonly called technology, is fixed. Hence the shift parameters are assumed to be constant.

#### **Technology Parameter Closure**

$$ADX_r = \overline{ADX0_r} \tag{C4.3}$$

$$ADVA_{r} = \overline{ADVA0_{r}} \tag{C4.4}$$

$$ADFD_f = \overline{ADFD0_f} \tag{C4.5}$$

Changes in efficiency can either be imposed exogenously or a technology variable can be freed up so as to satisfy some other condition, e.g., the required efficiency gain needed to satisfy a predetermined increase in GDP.

#### 5.5 Numéraire

The model specification allows for a choice between two price normalisation equations, the consumer price index (*CPI*) and a producer price index (*PPI/PD*). A *numéraire* is needed for © McDonald & Thierfelder



each region, to serve as a base since the model is homogenous of degree zero in prices for each region, and hence only defines relative prices.

#### Numéraire Closure

$$CPI_r = \overline{CPI_r} * numerchk$$
 (C5a)

$$PD_r = \overline{PD0_r} * numerchk$$
 (C5b)

#### 5.7 Factor Market Closure

The factor market closure rules allow for either full employment or unemployment. If a factor is unemployed, there is a threshold wage and additional quantities of the factor can be employed without the wage increasing; however, when there are no longer unemployed factors, the wage must adjust as factor demand increases. The model specification is a mixed complimentary problem.

#### 5.7.1 Full Factor Mobility and Employment Closure

This factor market closure requires that the total supply of and total demand for factors equate, and since the total supplies of each factor are determined exogenously this condition is satisfied by the equilibrium condition (C6.1a). The demands for factor f by activity a in region r and the wage rates for factors are determined endogenously. Bounds are placed upon the average factor prices, (C6.1b), so that meaningful results are produced.

## Full Employment and Mobile Factors

$$FS_{f,r} = \overline{FS}_{f,r} \tag{C6.1a}$$

 $Min WF_{f,r} = 0$ 

$$Max WF_{f,r} = +infinity (C6.1b)$$



# **Concluding Comments**

This paper has described a new global CGE model - R23. The R23 model described here contains relatively simple behavioural relationships although it requires a programme that contains a number of technically sophisticated features. Moreover, it is capable of being implemented with any aggregation of the 203 countries in the R23 database. This version of the model will be made generally available after further testing.



## **Appendices**

#### A1. R23 Model Genealogy

The R23 model project started in Oct 2013 with funding from the World Bank's KCP funds. It is a development of the 123 model, originally developed by Sherman Robinson and colleagues and subsequently developed as model for use by the World Bank by Deverajan *et al.*, (1997), and the GLOBE model.

The GLOBE model project started in June 2002 at a conference on *Poverty, Trade & Tools for Development*, which was hosted by the International Food Policy Research Institute in honour of Sherman Robinson's 60<sup>th</sup> birthday. During that conference Karen Thierfelder and Scott McDonald discussed, and agreed on, the creation of a SAM based global CGE model calibrated using the GTAP database. Karen and Scott had both 'learnt' CGE models starting with version of a single country CGE model developed for the US Department of Agriculture's (USDA) Economic Research Service (ERS) under the leadership of Sherman Robinson (Robinson *et al.*, 1990; Kilkenny, 1991). Karen had worked on the NAFTA project, also led by Sherman Robinson, and had subsequently used a global model that was descended from the NAFTA model. Scott had been introduced to GTAP by Terrie Walmsley and had used SAMs derived from the GTAP database for teaching single country CGE modelling. These teaching models and the PROVIDE project model (McDonald, 2003) provided the core single country code used to develop GLOBE.<sup>26</sup>

The first full versions of the R23 model were in use by mid-2013 (R23\_mod\_v1.gms) and formed the basis of a short course run prior to the GTAP conference in Daker. The first formal documentation (McDonald and Thierfelder, 2013) appeared at the same time. Version 1 of the R23 model used an aggregation of the GTAP database (version 8) together with additional data for remittance, transfers and aid.

The R23 database was developed during 2014 and 2015 with Terrie Walmsley taking the lead. This database does not rely on the published GTAP database but uses, *inter alia*,

The IFPRI standard model (Lofgren *et al.*, 2001) was also a descendant of the USDA's ERS model. Both Karen and Scott were in close contact with the team at IFPRI during the development of the IFPRI standard model and aspects of that model have clearly influenced both the GLOBE and STAGE models.

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World Bank, IMF and GTAP data to develop a new database that covers 203 countries/regions (see McDonald *et al.*, 2016b) for details). The R23 database provides the data for the revised and extended R23 model (R23\_2.gms) was completed in late 2015, and is documented in McDonald and Thierfelder., (2016a). The R23 model and database were made open source in early 2016.

The R23 model is part of a suite of models that include a single country model (STAGE – a development of the PROVIDE model that was developed from the USDA ERS model), a global model (GLOBE) and a range of teaching models – the SMOD suite. All these models use a (overwhelmingly) common set of notation and formats.



## **A2** Equation and Variable Counts for the R23 CGE Model

| Name              | Equation  | Number of<br>Equations | Variable    | Number of<br>Variables | Globe |
|-------------------|---|------------------------|-------------|------------------------|-------|
|                   | EXPORTS BLOCK   |                        |             |                        |       |
| $PEDEF_r$         | $PE_{r} * QE_{r} = \sum_{w} PER_{w,r} * QER_{w,r} $ $\forall r \in rgn$   | (rgn)                  | $PE_r$      | (rgn)                  | NO    |
| $PERDEF2_{w,r}$   | $PER_{w,r} = PWE_{w,r} * ER_r * (1 - TE_{w,r}) $ $\forall r \in rer$  | (w*r)                  | $PER_{r,w}$ | (w*r)                  | NO    |
| $PERDEFHG_{,w,r}$ | $PER_{w,r} = PE_r  \forall r \notin rgn, w \in wgn$   | (w*1)                  |             | ( <i>w</i> *1)         | YES   |
| $CET_r$           | $QX_{r} = at_{r} \cdot \left(\gamma_{r} * QE_{r}^{\rho_{r}^{l}} + (1 - \gamma_{r}) * QD_{r}^{\rho_{r}^{l}}\right)^{\frac{1}{\rho_{r}^{l}}}$ $\forall r \in (rd \cap re), r \in rgn$ | (rgn)                  | $QD_r$      | (rgn)                  | NO    |
| $ESUPPLY_r$       | $QE_r = QD_r \left[ \frac{PE_r}{PD_r} * \frac{(1 - \gamma_r)}{\gamma_r} \right]^{\frac{1}{(\rho_r' - 1)}}  \forall r \in (rd \cap re), r \in rgn$                                   | (rgn)                  | $QE_r$      | (rgn)                  | NO    |
| $CETALT_r$        | $QX_r = QD_r + QE_r$ $\forall (r \in (rd \cap ren), r \in rgn)$<br>$OR \ \forall (r \in (rdn \cap re), r \in rgn)$  |                        |             |                        | NO    |



| Name            | Equation   | Number of<br>Equations | Variable    | Number of<br>Variables | Globe |
|-----------------|--|------------------------|-------------|------------------------|-------|
|                 | EXPORTS BLOCK  |                        |             |                        |       |
| $CETLEV2_{w,r}$ | $QER_{w,r} = QE_r * \left(\frac{PER_{w,r}}{\left(\left(PE_r * \gamma_{w,r}^r * at_r^r\right)^{\rho_r^e}\right)}\right)^{\left(\sqrt{\left(\rho_r^e - 1\right)}\right)}$ $\forall r \in rgn \text{ and } r \in rer$ | (rgn*w)                | $QER_{r,w}$ | (rgn*w)                | NO    |
| $PERDEF2_{w,r}$ | $QE_r = \sum_{w} QER_{w,r} \qquad \forall r \notin rgn$  | (1)                    | $QE_r$      | (1)                    | YES   |
| $TSHIP_{,r}$    | $QE_r = QM_r$ $\forall r \notin rgn$   | (1)                    | $QE_r$      | (1)                    | YES   |



R23 Model

| Name  | Equation  |                     | Number of<br>Equations | Variable     | Number of<br>Variables | Globe |
|---|---|---------------------|------------------------|--------------|------------------------|-------|
|   | IMPORTS BLOCK   |                     |                        |              |                        |       |
| $PMRDEF2_{w,r}$   | $PMR_{w,r} = PWM_{w,r} * ER_r * (1 + TM_{w,r})$   | $\forall r \in rmr$ | (r*w)                  | $PMR_{w,,r}$ | (r*w)                  | YES   |
| $PMDEF_r$   | $PM_{r} * QM_{r} = \sum_{w \text{$\varsigma$cmr}_{w,r}} \left( PMR_{w,r} * QMR_{w,r} \right)$   | $orall rm_r$       | ( <i>r</i> )           | $PM_r$       | <i>(r)</i>             | YES   |
| $PWMDEF_{w,r}$  | $PWM_{w,r} = PWMFOB_{w,r} + \left(margcor_{w,r} * PT_r\right)$  | $\forall r \in rmr$ | (r*w)                  | $PWM_{r,w}$  | (r*w)                  | YES   |
| $ARMINGTON_r$   | $QQ_r = ac_r \left( \delta_r * QM_r^{-\rho_r^c} + (1 - \delta_r) * QD_r^{-\rho_r^c} \right)^{-\left(\frac{1}{\rho_r^c}\right)}$   | C 10100             | ( <i>r</i> )           | $QQ_r$       | (r)                    | YES   |
| $COSTMIN_r$   | $QM_{r} = QD_{r} * \left( \left( \frac{PD_{r}}{PM_{r}} \right) * \left( \frac{\delta_{r}}{(1 - \delta_{r})} \right) \right)^{\frac{1}{(1 + \rho_{r}^{c})}}  \forall r \in \mathbb{R}$ |                     | (r)                    | $QM_r$       | <i>(r)</i>             | YES   |
| $ARMALT_r$ $QQ_r = QD_r + QM_r$ $\forall r \in rgn, r \in rx, r \in rm$ |   |                     |                        |              |                        |       |
| $ARMLEV3_r$   | $QMR_{w,r} = QML_r * \left(\frac{PMR_{w,r} * acr_r^{\rho_r^m}}{PM_r * \delta_{w,r}^r}\right)^{\left(-1/(\rho_r^m + 1)\right)}$  | $\forall r \in rmr$ | (r)                    | $QML_r$      | <i>(r)</i>             | YES   |
| $QTEQ_{w,r}$  | $QT_{w,r} = QMR_{w,r} * margcor_{w,r} \qquad \forall r \in rgn$   |                     | (w*r)                  | $QT_{w,r}$   | (w*r)                  | YES   |



| Name                  | Equation  | Number of<br>Equations | Variable    | Number of<br>Variables | Globe |
|-----------------------|---|------------------------|-------------|------------------------|-------|
|                       | COMMODITY PRICE BLOCK   |                        | PD          |                        |       |
| $PQSDEF_r$            | $PQS_r = \frac{\left(PD_r * QD_r\right) + \left(PM_r * QM_r\right)}{QQ_r} \qquad \forall r \in rgn$ | (rgn)                  | $PQS_{c,r}$ | (rgn)                  | NO    |
| $PQDDEF_r$            | $PQD_r = PQS_r * (1 + TS_r) \qquad \forall r \in rgn$   | (rgn)                  | $PQD_{c,r}$ | (rgn)                  | NO    |
|                       | NUMERAIRE PRICE BLOCK   |                        |             |                        |       |
| $\overline{CPIDEF_r}$ | $CPI_r = PQD_r * (1 + TV_r) $ $\forall r \in rgn$   | rgn                    | $CPI_r$     | rgn                    | NO    |
| ERPIDEF               | $ERPI = \sum_{ref} vqesh_{ref} * ER_{ref}$  | 1                      | ERPI        | 1                      | NO    |



R23 Model

| Name          | Equat   | ion  | Number of<br>Equations | Variable     | Number of Variables | Globe |
|---------------|---|--|------------------------|--------------|---------------------|-------|
|               | PRODUCTION BLOC   |  |                        |              |                     |       |
| $PXDEF_r$     | $PX_{r} = \frac{\left(PD_{r} * QD_{r}\right) + \left(PE_{r} * QE_{r}\right)}{QX_{r}}$   | $\forall r \in rgn, r \in rx$  | (rgn)                  | $PX_{a,r}$   | (rgn)               | NO    |
| $PVADEF_r$    | $PX_r * (1 - TX_r) * QX_r = (PVA_r * QVA$   | $(r) + (PQD_r * QINT_r)  \forall r \in rgn$  | (rgn)                  | $PVA_{a,r}$  | (rgn)               | NO    |
| $QXPRODFN_r$  | $QX_r = ADX_r * \left[ \delta_r^x * (QVA_r)^{-\rho_r^x} + (1 + \epsilon)^{-\rho_r^x} \right]$   | $\forall r \in rgn, r \in rqx$   | (rgn)                  | $QX_{a,r}$   | (rgn)               | NO    |
| $QXFOC_{f,r}$ | $QVA_{r} = QINT_{r} * \left( \left( \frac{PQD_{r}}{PVA_{r}} \right) * \left( \frac{\delta_{r}^{x}}{\left( 1 - \delta_{r}^{x} \right)} \right) \right) $ | $\left(\frac{1}{(1+\rho_r^x)}\right) \sqrt{\left(1+\rho_r^x\right)} \qquad \forall r \in rgn, r \in rqx$ | (rgn)                  | $QINT_{a,r}$ | (rgn)               | NO    |
| $QINTDEF_r$   | $QINT_r = ioqintqx_r * QX_r$  | $\forall r \in rgn, r \in rqxn$  | (rgn)                  | $QX_{a,r}$   | (rgn)               | NO    |
| $QVADEF_r$    | $QVA_r = ioqvaqx_r * QX_r$  | $\forall r \in rgn, r \in rqxn$  | (rgn)                  | $QINT_{a,r}$ | (rgn)               | NO    |



| Name           | Equation  | Number of<br>Equations    | Variable             | Number of<br>Variables | Globe |
|----------------|---|---------------------------|----------------------|------------------------|-------|
|                | PRODUCTION BLOCK  |                           |                      |                        |       |
| $QVAPRODFN_r$  | $QVA_{r} = ADVA_{r} * \left( \sum_{f} \delta_{f,r}^{va} * \left( ADFD_{f,r} * FD_{f,r} \right)^{-\rho_{r}^{va}} \right)^{\left(-\frac{1}{\rho_{r}^{va}}\right)}  \forall r \in rgn$   | (rgn)                     | $\mathit{QVA}_{a,r}$ | (rgn)                  | NO    |
| $QVAFOC_{f,r}$ | $WF_{f,r} * (1 + TF_{f,r}) = PVA_{r} * QVA_{r} * \left[ \sum_{f} \delta_{f,r}^{va} * (ADFD_{f,r} * FD_{f,r})^{-\rho_{r}^{va}} \right]^{(r)} \\ * \delta_{f,r}^{va} * (ADFD_{f,r})^{-\rho_{r}^{va}} * (FD_{f,r})^{(-\rho_{r}^{va}-1)} \\ \forall r \in rgn, \delta_{f,r}^{va} > 0$ | ( <i>f</i> * <i>rgn</i> ) | $FD_{a,f,r}$         | (f*rgn)                | NO    |



| Name              | Equation  | Number of<br>Equations | Variable        | Number of<br>Variables | Globe |
|-------------------|---|------------------------|-----------------|------------------------|-------|
|                   | FACTOR BLOCK  |                        |                 |                        |       |
| $YFEQ_{f,r}$      | $YF_{f,r} = \sum_{a} WF_{f,r} * FD_{f,r} \qquad \forall r \in rgn$  | (f*rgn)                | $YF_{f,r}$      | (f*rgn)                | NO    |
| $YFOUTEQ_{w,f,r}$ | $YFOUT_{w,f,r} = yfoutsh_{w,f,r} * (YF_{f,r} * (1 - deprec_{f,r}))  \forall r \in rgn, w \in w$   | egn (w*f*r)            | $YFOUT_{w,f,r}$ | (w*f*r)                | NO    |
| $YFINEQ_{f,w,r}$  | $YFIN_{f,w,r} = \left(\sum_{w\$map\_r\_w;r\$map\_w\_r} \frac{YFOUT_{w,f,r}}{ER_r}\right) * ER_r  \forall r \in rgn, w \in w$  | ygn<br>(f*w*r)         | $YFIN_{f,w,r}$  | $(f^*w^*r)$            | NO    |
| $YFDISTEQ_{f,r}$  | $YFDIST_{f,r} = \left( \left( YF_{f,r} * \left( 1 - deprec_{f,r} \right) \right) + \sum_{w} YFIN_{f,w,r} - \sum_{w} YFOUT_{w,f,r} \right) * \left( 1 - TYF_{f,r} \right)$ | ∀r∈ <b>t§n</b> gn)     | $YFDIST_{f,r}$  | (f*rgn)                | NO    |



| Name                    | Equation   | Number of<br>Equations | Variable      | Number of<br>Variables | Globe |
|-------------------------|--|------------------------|---------------|------------------------|-------|
|                         | HOUSEHOLD BLOCK  |                        |               |                        |       |
| $YHEQ_{h,r}$            | $YH_r = \sum_{f} hvash_{f,r} *YFDIST_{f,r} + (HGADJ * hogovconst_r * CPI_r)$   | (h*rgn)                | $YH_{h,r}$    | (h*rgn)                | NO    |
| $IIIL\mathcal{Q}_{h,r}$ | $\forall r \in rgr$  |                        | h,r           | (n rgn)                | NO    |
| $REMITOUTEQ_{w,h,r}$    | $REMITOUT_{w,h,r} = remitoutsh_{w,h,r} * (YH_r * (1 - TYH_r))$   |                        |               |                        |       |
| TELITITO OTE E W, II, I | $\forall r \in rgn, w \in wgn$   |                        |               |                        |       |
| $REMITIN_{h,w,r}$       | $REMITIN_{h,w,r} = \left(\sum_{w\$map\_r\_w;r\$map\_w\_r} \frac{REMITOUT_{w,h,r}}{ER_r}\right) * ER_r$   |                        |               |                        |       |
|                         | $\forall r \in rgn, w \in wgn$   |                        |               |                        |       |
| $QCDEQ_{h,r}$           | $QCD_{r} * PQD_{r} * (1 + TV_{r}) = \begin{pmatrix} (YH_{r} * (1 - TYH_{r})) \\ -\sum_{w,h} REMITOUT_{w,h,r} \\ +\sum_{h,w} REMITIN_{h,w,r} \end{pmatrix} * (1 - SHH_{r})$ | (h*rgn)                | $QCD_{c,h,r}$ | (h*rgn)                | NO    |
|                         | $\forall r \in rgr$  | $\imath$               |               |                        |       |



| Name        | Equation   |                                      | Number of Equations | Variable   | Number of<br>Variables | Globe |
|-------------|--|--------------------------------------|---------------------|------------|------------------------|-------|
|             | GOVERNMENT TAXES BLOCK   |                                      |                     |            |                        | _     |
| $MTAXEQ_r$  | $MTAX_{r} = \sum_{w} \left( TM_{w,r} * PWM_{w,r} * ER_{r} * QMR_{w,r} \right)$                                     | $\forall r \in rgn, r \in rm$        | rgn                 | $MTAX_r$   | rgn                    | NO    |
| $ETAXEQ_r$  | $ETAX_{r} = \sum_{w} \left( TE_{w,r} * PWE_{w,r} * ER_{r} * QER_{w,r} \right)$                                     | $\forall r \in rgn, r \in re$        | rgn                 | $ETAX_r$   | rgn                    | NO    |
| $STAXEQ_r$  | $STAX_r = (TS_r * PQS_r * (QINT_r + QCD_r + QGD_r + QIN))$ $\forall r \in rgn, 0$                                  | $(VD_r)$ $(r \in re \text{ or } rm)$ | rgn                 | $STAX_r$   | rgn                    | NO    |
| $VTAXEQ_r$  | $VTAX_r = (TV_r * PQD_r * QCD_r)$ $\forall r \in rg$   | $gn, (r \in re \text{ or } rm)$      | rgn                 | $INDTAX_r$ | rgn                    | NO    |
| $ITAXEQ_r$  | $ITAX_{r} = (TX_{r} * PX_{r} * QX_{r})$  | $\forall r \in rgn$                  | rgn                 | $INDTAX_r$ | rgn                    | NO    |
| $FYTAXEQ_r$ | $FYTAX_{r} = \sum_{f} \left( TYF_{f,r} * \left( YF_{f,r} - \left( deprec_{f,r} * YF_{f,r} \right) \right) \right)$ | $\forall r \in rgn$                  | rgn                 | $FYTAX_r$  | rgn                    | NO    |
| $HTAXEQ_r$  | $HTAX_r = (TYH_r * YH_r)$  | $\forall r \in rgn$                  | rgn                 | $HTAX_r$   | rgn                    | NO    |
| $FTAXEQ_r$  | $FTAX_{r} = \sum_{f} \left( TF_{f,r} * WF_{f,r} * FD_{f,r} \right)$  | $\forall r \in rgn$                  | rgn                 | $FTAX_r$   | rgn                    | NO    |



| Name        | Equation   | Number of<br>Equations | Variable  | Number of<br>Variables | Globe |
|-------------|--|------------------------|-----------|------------------------|-------|
| GO          | OVERNMENT INCOME AND EXPENDITURE BLOCK   |                        |           |                        |       |
|             | $YG_r = MTAX_r + ETAX_r + STAX_r + VTAX_r + ITAX_r$  |                        |           |                        |       |
| $YGEQ_r$    | $+FYTAX_{r} + HTAX_{r} + FTAX_{r}$   | rgn                    | $YG_r$    | rgn                    | NO    |
|             | $+\sum_{g,w}AID\_B\_IN_{g,w,r} + \sum_{g,w}AID\_M\_IN_{g,w,r}  \forall r$                                    |                        |           |                        |       |
| $EGEQ_r$    | $EG_{r} = PQD_{r} * QGD_{r} + (HGADJ_{r} * hogovconstr * CPI_{r})$ $\forall r \in rgn$                       | rgn                    | $EG_r$    | rgn                    | NO    |
| AID_B_OUTEQ | $AID\_B\_OUT_{w,g,r} = aid\_b\_outsh_{w,g,r} *GDP_r$<br>$\forall r \in rgn, w \in wgn, g \in gt$             | (rgn*wgn)              | AID_B_OUT | (rgn*wgn)              | NO    |
| AID_B_INEQ  | $AID\_B\_IN_{g,w,r} = \left(\sum_{w\$map\_r\_w,r\$map\_w\_r} \frac{AID\_B\_OUT_{w,g,r}}{ER_r}\right) * ER_r$ | (rgn*wgn)              | AID_B_IN  | (rgn*wgn)              | NO    |
|             | $\forall r \in rgn, w \in wgn, g \in$  | gt                     |           |                        |       |
|             | $AID\_M\_OUT_{w,g,r} = aid\_m\_outsh_{w,g,r} *GDP_r  \forall r \in rgn, g \in gu$                            | t                      |           |                        |       |
|             | $AID\_M\_OUT_{w,g,r} = aid\_m\_outsh_{w,g,r} * YG_r  \forall r \notin rgn, g \in g$                          | t                      |           |                        |       |
|             | $AID\_M\_IN_{g,w,r} = \left(\sum_{w\$map\_r\_w;r\$map\_w\_r} \frac{AID\_M\_OUT_{w,g,r}}{ER_r}\right) * ER_r$ |                        |           |                        |       |
|             | $\forall g \in gt$   |                        |           |                        |       |



| Name         | Equation  |                     | Number of<br>Equations | Variable   | Number of<br>Variables | Globe |
|--------------|---|---------------------|------------------------|------------|------------------------|-------|
|              | KAPITAL ACCOUNT BLOCK   |                     |                        |            |                        |       |
| $TOTSAVEQ_r$ | $TOTSAV_{r} = \left( \left( YH_{r} * (1 - TYH_{r}) \right) - \sum_{w,h} REMITOUT_{w,h,r} + \sum_{h,w} RH_{r} + \sum_{f} \left( deprec_{f,r} * YF_{f,r} \right) \right)$ | ,                   | rgn                    | $TOTSAV_r$ | rgn                    | NO    |
|              | $+KAPGOV_r + (KAPWOR_r)$  | $*ER_r$ )           | $\forall r \in rgn$    |            |                        |       |
| $INVESTEQ_r$ | $INVEST_r = (PQD_r * QINVD_r)$  | $\forall r \in rgn$ | rgn                    | $INVEST_r$ | rgn                    | NO    |



| Name                                 | Equation  |                           | Number<br>of<br>Equations | Variable               | Number<br>of<br>Variables | Globe |
|--------------------------------------|---|---------------------------|---------------------------|------------------------|---------------------------|-------|
|                                      | MARKET CLEARING BLOCK   |                           |                           |                        |                           |       |
| $FMEQUIL_{f,r}$                      | $FS_{f,r} = FD_{f,r} + UNEMP_{f,r} \qquad \forall r \in rgn$  |                           | (f*rgn)                   | $FS_{f,r}$             | (f*rgn)                   | NO    |
| $\mathit{UNEMPEQUIL}_{\mathit{f,r}}$ | $UNEMP_{f,r} \ge 0$ $\forall r \in rgn$   |                           | (f*rgn)                   | $\mathit{UNEMP}_{f,r}$ | (f*rgn)                   | NO    |
| $QEQUIL_{c,r}$                       | $QQ_r = QINT_r + QCD_r + QGD_r + QINVD_r$   | $\forall r \in rgn$       | (rgn)                     | $QQ_r$                 | (rgn)                     | NO    |
| $COMTRADE_r$                         | $\sum_{w} \left( PWM_{w,r} * QMR_{w,r} \right) = \sum_{w} \left( PWE_{w,r} * QER_{w,r} \right) + GLC$ | $OBESLACK  \forall r \in$ | <b>≠</b> 1                | GLOBESLACK             | 1                         | YES   |
| $TRCONP_{w,r}$                       | $PWMFOB_{w,r} = PWE_{w,r}$  |                           | (r*w)                     | $PWMFOB_{r,w}$         | (r*w)                     | YES   |
| $TRCONQ_{w,r}$                       | $QMR_{w,r} = QER_{w,r}$   |                           | (r*w)                     | $QMR_{r,w}$            | (r*w)                     | YES   |
| $AIDTRADE_r$                         | $\sum_{w} AID \_M \_OUT_{w,g,r} = \left(\sum_{w} AID \_M \_IN_{g,w,r}\right) + G$                     | GLOBESLACK3               | $\forall r \notin rgn$    |                        |                           |       |
| $GLOBEQUIL_r$                        | $\sum_{w} QT_{w,r} = \sum_{w} QER_{w,"glo"} \qquad \forall r \in rgn$                                 |                           | (rgn)                     |                        | (rgn)                     | YES   |
| $PTDEF_r$                            | $PT_r = PWE_{w,"glo"} \qquad \forall r \in rgn$   |                           | (rgn)                     | $PT_r$                 | (rgn)                     | YES   |
| $GLOBEQUIL_r$                        | $\sum_{w} QT_{w,r} = \sum_{w} QER_{w,"glob"} \qquad \forall r \in rgn$                                |                           |                           |                        |                           |       |



|                   |  | Number                    |                | Number          | G1.1  |
|-------------------|--|---------------------------|----------------|-----------------|-------|
| Name              | Equation   | of<br>Equations           | Variable       | of<br>Variables | Globe |
|                   | $MACROECONOMIC\ CLOSURES\ BLOCK\ KAPGOV_r = YG_r - EG_r$   | •                         |                |                 |       |
| $KAPGOVEQ_r$      | $-\sum_{w,g} AID\_B\_OUT_{w,g,r} - \sum_{w,g} AID\_M\_OUT_{w,g,r}$   | $\forall r \in rgn^{rgn}$ | $KAPGOV_r$     | rgn             | NO    |
| $KAPREQUIL_{r,w}$ | $KAPREG_{w,r} = \begin{pmatrix} (PWMFOB_{w,r} * QMR_{w,r}) \\ + \sum_{f} \frac{YFOUT_{w,f,r}}{ER_{r}} + \sum_{h} \frac{REMITOUT_{w,h,r}}{ER_{r}} \\ + \sum_{g} \frac{AID\_B\_OUT_{w,g,r}}{ER_{r}} + \sum_{g} \frac{AID\_M\_OUT_{w,g,r}}{ER_{r}} \\ - \begin{pmatrix} (PWE_{w,r} * QER_{w,r}) \\ + \sum_{f} \frac{YFIN_{f,w,r}}{ER_{r}} + \sum_{h} \frac{REMITIN_{h,w,r}}{ER_{r}} \\ + \sum_{g} \frac{AID\_B\_IN_{g,w,r}}{ER_{r}} + \sum_{g} \frac{AID\_M\_IN_{g,w,r}}{ER_{r}} \end{pmatrix}$ |                           | $KAPREG_{r,w}$ | (r*wgn)         | NO    |



|                           |   | Number   |                | Number    |       |
|---------------------------|---|--|----------------|-----------|-------|
| Name                      | Equation  | of   | Variable       | of        | Globe |
|                           |   | Equations                                      |                | Variables |       |
|                           | MACROECONOMIC CLOSURES BLOCK  |  |                |           |       |
|                           | $KAPREG_{w,r} = \left( \left( \sum_{w} PT_r * QT_{w,r} \right) + \sum_{f} \frac{YFOUT_{w,f,r}}{ER_r} + \sum_{f} YFOU$ | $\sum_{g} \frac{AID\_M\_OUT_{w, g, r}}{ER_r} $ |                |           |       |
| KAPREQUIL2 <sub>r,w</sub> | $-\left(\left(PWE_{w,r} * QER_{w,r}\right) + \sum_{f} \frac{YFIN_{f,w,r}}{ER_{r}}\right)$   | $+\sum_{g}\frac{AID\_M\_IN_{g,w,r}}{ER_{r}}$   | $KAPREG_{r,w}$ | r         | YES   |
|                           |   | $\forall w \notin wgn$                         |                |           |       |
| $KAPEQUIL_r$              | $KAPWOR_r = \sum_{w} KAPREG_{w,r}$ $\forall r$  | r  | $KAPWOR_r$     | r         | YES   |



| Name           | Equation   |                     | Number<br>of<br>Equations | Variable     | Number<br>of<br>Variables | Globe |
|----------------|--|---------------------|---------------------------|--------------|---------------------------|-------|
|                | ABSORPTION CLOSURES BLOCK  |                     |                           |              |                           |       |
| $VFDOMDEQ_r$   | $VFDOMD_r = \left(PQD_r * \left(\left(QCD_r * \left(1 + TV_r\right)\right) + QC\right)\right)$ | $GD_r + QINVD_r$    | ∀rgn∈ rgn                 | $VFDOMD_r$   | rgn                       | NO    |
| $INVESTSHEQ_r$ | $INVESTSH_r * VFDOMD_r = INVEST_r$   | $\forall r \in rgn$ | rgn                       | $INVESTSH_r$ | rgn                       | NO    |
| $VGDSHEQ_r$    | $VGDSH_r * VFDOMD_r = (PQD_r * QGD_r)$   | $\forall r \in rgn$ | rgn                       | $VGDSH_r$    | rgn                       | NO    |
|                | $MACROECONOMIC AGGREGATES BLOCGDP_r = (PQD_r * (1+TV)_r * QCD_r) + (PQD_r * (1+TV)_r * QCD_r)$ | · <del></del>       |                           |              |                           |       |
|                | $+(PE_r*QE_r)-(PM_r*QM_r)$   | $\forall r \in rgn$ |                           |              |                           |       |
|                | SLACKS BLOCK   |                     |                           |              |                           |       |
|                | $TOTSAV_r = INVEST_r + WALRAS_r$   | $\forall r \in rgn$ |                           |              |                           |       |
|                | $KAPWORSYS = \sum_{r} KAPWOR_{r}$  | $\forall r \in rgn$ |                           |              |                           |       |



| Name | Equation  | Number of<br>Equations   | Variable  | Number of<br>Variables |
|------|---|--|---|------------------------|
|      | MODEL CLOSURE  All globe transactions are in world dollars (i.e. of the   |  |   |                        |
|      | reference region)   | $\mathit{ER}_{glo}$  |   |                        |
|      | There is no current account equation for the globe.   | AND $\overline{\mathit{KAPWOR}_{\mathit{glo}}}$                              |   |                        |
|      | Assign the reference region, eliminating one variable; add the constraint that the sum of KAPWOR across <i>rgn</i> must |  | $\overline{\it ER}_{\it rej}$                     | f                      |
|      | be zero   | KAPWORSYS  |   |                        |
|      | For the non-reference regions, one can fix either the exchange rate or the current account balance                      | $\overline{ER}_{nonref}$ or  | KAPWOR <sub>nonref</sub>                          | <del>-</del>           |
|      |   | $\overline{SADJ}$ or $\overline{IAD}$  | $\overline{OJ}$ or $\overline{INVEST}$            | -                      |
|      | All <b>but</b> two of $\overline{TM}$ , $\overline{TS}$ , $\overline{T}$  | $\overline{V}, \overline{TE}, \overline{TX}, \overline{TYH}, \overline{TYF}$ | $\overline{TF}, \overline{KAPGOV}, \overline{EC}$ | ř                      |
|      |   |  | $\overline{FS}$ ,                                 | f                      |
|      |   |  | $\overline{CPI}$ or $\overline{PPI}$              | <u></u>                |



## References

- de Melo, J. and Robinson, S., (1985). 'Product Differentiation and Trade Dependence of the Domestic Price System in Computable General Equilibrium Trade Models', in Peeter, T., Prate, P. and Reding, P., (eds) *International Trade and Exchange Rates in the Late Eighties*. Amsterdam: North-Holland.
- de Melo, J. and Robinson, S., (1989). 'Product Differentiation and the Treatment of Foreign Trade in Computable General Equilibrium Models of Small Economies', *Journal of International Economics*, Vol 27, pp 47-67.
- Devarajan, S., Lewis, J.D. and Robinson, S., (1990). 'Policy Lessons from Trade-Focused, Two-Sector Models', *Journal of Policy Modeling*, Vol 12, pp 625-657.
- Devarajan, S., Go, D.S., Lewis, J.D., Robinson, S. and Sinko, P., (1997). 'Simple General Equilibrium Modeling', in Francois, J.F. and Reinert, K.A., (eds) *Applied Methods for Trade Policy Analysis: A Handbook*. Cambridge: Cambridge University Press.
- Drud, A., Grais, W. and Pyatt, G., (1986). 'Macroeconomic Modelling Based on Social-Accounting Principles', *Journal of Policy Modeling*, Vol 8, pp 111-145.
- Hertel, T.W., (1997). *Global Trade Analysis: Modeling and Applications*. Cambridge: Cambridge University Press.
- Kilkenny, M., (1991). Computable General Equilibrium Modelling of Agricultural Policies: Documentation of the 30-Sector FPGE GAMS Model of the United States. *USDA ERS Staff Report AGES 9125*.
- Kilkenny, M. and Robinson, S., (1990). 'Computable General Equilibrium Analysis of Agricultural Liberalisation: Factor Mobility and Macro Closure', *Journal of Policy Modeling*, Vol 12, pp 527-556.
- Lewis, J., Robinson, S., and Wang, Z. (
- Löfgren, H., Harris, R.L. and Robinson, S., with Thomas, M. and El-Said, M., (2002).

  Microcomputers in Policy Research 5: A Standard Computable General Equilibrium (CGE) Model in GAMS. Washington: IFPRI.
- McDonald, S. (2011).
- McDonald, S. and Sonmez, Y., (2004). 'Augmenting the GTAP Database with Data on Inter-Regional Transactions', *Sheffield Economics Research Paper 2004:009*. The University of Sheffield
- McDonald, S., Thierfelder, K. and Robinson, S., (2007). 'Globe v1: A SAM Based Global CGE Model using GTAP Data', USNA Working Paper no. 14.
- McDonald, S. and Thierfelder, K., (2016). 'R23 Model: A SAM Based Global Macro CGE Model.' Mimeo.



- McDonald, S., Thierfelder, K. and Walmsley, T., (2016). 'R23 Database.' Mimeo.
- McDonald, S. and Thierfelder, K., (2013). 'Globe v2: A SAM Based Global CGE Model using GTAP Data', mimeo. (www.cgemod.org.uk)
- McDonald, S., (2003). 'A Standard Computable General Equilibrium Model: Technical Documentation', *PROVIDE Project Technical Paper 2003:03*. Elsenburg, RSA.
- McDonald, S. and Tlhalefang, J., (2001). 'CGE Models of Energy Use: A Review of Production Structures and Data Requirements', *Report for the RANESA Project 'Energy Accounts for Botswana'*.
- McDonald, S., and Thierfelder, K., (2004a). 'Deriving a Global Social Accounting Matrix from GTAP version 5 Data', *GTAP Technical Paper* 23. Global Trade Analysis Project: Purdue University.
- McDonald, S., and Thierfelder, K., (2004b). 'Deriving Reduced Form Global Social Accounting Matrices from GTAP Data', *mimeo*.
- McDonald, S. and Sonmez, Y., (2006), 'Labour Migration and Remittances: Some Implications of Turkish "Guest Workers" in Germany', 9th Annual Conference on Global Economic Analysis, UN Economic Commission for Africa, Addis Ababa, Ethiopia, June.
- McDonald, S., and Thierfelder, K., (2006). 'The Doha Development Agenda and Africa:

  Taking Armington Seriously', *9th Annual Conference on Global Economic Analysis*,

  UN Economic Commission for Africa, Addis Ababa, Ethiopia, June.
- PROVIDE (2004a). 'SeeResults: A Spreadsheet Application for the Analysis of CGE Model Results', *PROVIDE Technical Paper*, 2004:1. Elsenburg, RSA.
- PROVIDE (2004b). 'SAMGator: A Spreadsheet Application for the Aggregation of Social Accounting Matrices', *PROVIDE Technical Paper*, forthcoming. Elsenburg, RSA.
- Pyatt, G., (1987). 'A SAM Approach to Modelling', *Journal of Policy Modeling*, Vol 10, pp 327-352.
- Pyatt, G., (1991). 'Fundamentals of Social Accounting', *Economic Systems Research*, Vol 3, pp 315-341.
- Robinson, S., (2004). 'Exchange Rates in Global CGE Models', paper presented at the *IIOA* and *EcoMod Conference*, Brussels, Sept 2004.
- Robinson, S., Burfisher, M.E., Hinojosa-Ojeda, R. and Thierfelder, K.E., (1993). 'Agricultural Policies and Migration in a US-Mexico Free Trade Area: A Computable General Equilibrium Analysis', *Journal of Policy Modeling*, Vol 15, pp 673-701.
- Robinson, S., Kilkenny, M. and Hanson, K., (1990). 'USDA/ERS Computable General Equilibrium Model of the United States', Economic Research Services, USDA, *Staff Report AGES 9049*.



- Sen, A.K., (1963). 'Neo-classical and Neo-Keynesian Theories of Distribution', *Economic Record*, Vol 39, pp 53-64.
- Stone, R., (1962a). 'A Computable Model of Economic Growth', *A Programme for Growth: Volume 1*. Cambridge: Chapman and Hall.
- Stone, R., (1962b). 'A Social Accounting Matrix for 1960', *A Programme for Growth: Volume 2*. Cambridge: Chapman and Hall.
- Tlhalefang, J., (2006). Computable General Equilibrium Implications of Oil Shocks and Policy Responses in Botswana, unpublished PhD thesis, Department of Economics, University of Sheffield.
- UN, (1993). System of National Accounts 1993. New York: UN.
- Walmsley, T., Ahmed, S.A. and Parsons, C., (2005). 'The GMig2 Data Base: A Data Base of Bilateral Labor Migration, Wages and Remittances', *GTAP Research Memorandum No.* 6.