



Social Accounting Matrix Multipliers




1

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Outline

- Introduction
- Materials Balance equation
- SAM Income Multipliers
 - Multipliers and Income Distribution
- SAM Price Multipliers
- Mixed Multipliers



2

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Material Balance Equation

$$\mathbf{q} = \mathbf{Aq} + \mathbf{d}$$

$$\mathbf{q} - \mathbf{Aq} = \mathbf{d}$$

$$\mathbf{q} = [\mathbf{I} - \mathbf{A}]^{-1} \mathbf{d}$$

Gross outputs (\mathbf{q}) is the sum of endogenously determined output (\mathbf{Aq}) plus exogenously determined output (\mathbf{d})

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Schematic SAM


			Expenditures							
			Endogenous accounts				Exogenous accounts			Totals
			1	2	3	4				
			Commodities	Activities	Factors	Households	Government	Investment	Rest of World	
Incomes	Endogenous accounts	1	Commodities	N_{11}	$\mathbf{0}$	N_{13}	x_1			y_1
			Activities							
		2	Factors	N_{21}	$\mathbf{0}$	$\mathbf{0}$	x_2			y_2
	3	Households	$\mathbf{0}$	N_{32}	N_{33}	x_3			y_3	
	Exogenous accounts	4	Government				t			y_4
			Investment	I'_1	I'_2	I'_3				
			Rest of World							
5	Totals	y'_1	y'_2	y'_3	y'_4					

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Notation

Expenditures	Exogenous Accounts	Totals
Endogenous Accounts $N = A_n \hat{y}_n \rightarrow (2)$	X	$y_n = n + x$ $= A_n y_n + x \rightarrow (4)$
Exogenous Accounts $L = A_l \hat{y}_n \rightarrow (3)$	R	$y_x = l + Ri$ $= A_l y_n + Ri \rightarrow (5)$
Totals $y'_n = (i'A_n + i'A_l) \hat{y}_n \rightarrow (6)$	$y'_x = i'X + i'R \rightarrow (8)$	
$\therefore i' = i'A_n + i'A_l \rightarrow (7)$	$\therefore A_l y_n - X i = (R - R') i \rightarrow (9)$	$\lambda'_n y_n = x i \rightarrow (10)$

 5


Income Multipliers

$$y_n = A_n y_n + x$$

$$y_n - A_n y_n = x$$

$$(I - A_n) y_n = x$$

$$y_n = (I - A_n)^{-1} x = M_a x$$

 6


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Income Multipliers

$$\begin{aligned} \mathbf{l} &= \mathbf{A}_l \mathbf{y}_n \\ &= \mathbf{A}_l (\mathbf{I} - \mathbf{A}_n)^{-1} \mathbf{x} = \mathbf{A}_l \mathbf{M}_a \mathbf{x} \end{aligned}$$

$$\left[(\mathbf{i}' \mathbf{A}_n + \mathbf{i}' \mathbf{A}_l) \hat{\mathbf{y}}_n \right]' \mathbf{i} = \mathbf{y}'_n \mathbf{i} = \mathbf{i}' (\mathbf{A}_n \mathbf{y}_n + \mathbf{x}) = \mathbf{i}' \mathbf{y}_n$$

LEAKAGES = INJECTIONS

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Interpretation

$$\mathbf{M}_a = (\mathbf{I} - \mathbf{A}_n)^{-1} = \mathbf{I} + \mathbf{A}_n + \mathbf{A}_n^2 + \mathbf{A}_n^3 + \mathbf{A}_n^4 + \dots$$

$$\begin{aligned} \mathbf{y} &\approx (\mathbf{I} + \mathbf{A}_n + \mathbf{A}_n^2 + \mathbf{A}_n^3 + \mathbf{A}_n^4 + \dots) \mathbf{x} \\ &= \mathbf{x} + \mathbf{A}_n \mathbf{x} + \mathbf{A}_n^2 \mathbf{x} + \mathbf{A}_n^3 \mathbf{x} + \mathbf{A}_n^4 \mathbf{x} + \dots \end{aligned}$$

$$\mathbf{A}_n > \mathbf{A}_n^2 > \mathbf{A}_n^3 > \mathbf{A}_n^4 > \dots$$


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Interpretation

$$\Delta \mathbf{y} \approx \Delta \mathbf{x} + \mathbf{A}_n \Delta \mathbf{x} + \mathbf{A}_n^2 \Delta \mathbf{x} + \mathbf{A}_n^3 \Delta \mathbf{x} + \mathbf{A}_n^4 \Delta \mathbf{x} + \dots$$

$\Delta \mathbf{x}$ = endogenous shock
 $\Delta \mathbf{y}$ = change in gross output


9


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Multipliers and Income Distribution

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{0} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix}$$

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{A}_{11} \mathbf{y}_1 + \mathbf{A}_{13} \mathbf{y}_3 + \mathbf{x}_1 \\ \mathbf{y}_2 &= \mathbf{A}_{21} \mathbf{y}_1 + \mathbf{x}_2 \\ \mathbf{y}_3 &= \mathbf{A}_{32} \mathbf{y}_2 + \mathbf{A}_{33} \mathbf{y}_3 + \mathbf{x}_3 \end{aligned}$$


10

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Multipliers and Income Distribution

$$\mathbf{y}_1 = \mathbf{A}_{11}\mathbf{y}_1 + \mathbf{A}_{13}\mathbf{y}_3 + \mathbf{x}_1$$

$$\mathbf{y}_1 - \mathbf{A}_{11}\mathbf{y}_1 = \mathbf{A}_{13}\mathbf{y}_3 + \mathbf{x}_1$$

Production component


$$\mathbf{y}_1 = (\mathbf{I} - \mathbf{A}_{11})^{-1} (\mathbf{A}_{13}\mathbf{y}_3 + \mathbf{x}_1)$$

$$\mathbf{q} = [\mathbf{I} - \mathbf{A}]^{-1} \mathbf{d}$$

$$\mathbf{q} = \mathbf{y}_1$$

$$(\mathbf{I} - \mathbf{A})^{-1} = (\mathbf{I} - \mathbf{A}_{11})^{-1}$$

$$\mathbf{d} = (\mathbf{A}_{13}\mathbf{y}_3 + \mathbf{x}_1)$$



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Price Multiplier

	1	2	3	4	5
Production	$\hat{p}_1' Q_{11}$	0	$\hat{p}_1' Q_{13}$	$\hat{p}_1' Q_{14}$	$\hat{p}_1' q_1$
Factors	$\hat{p}_2' Q_{21}$	0	0	$\hat{p}_2' Q_{24}$	$\hat{p}_2' q_2$
Households	0	$\hat{p}_3' Q_{32}$	$\hat{p}_3' Q_{33}$	$\hat{p}_3' Q_{34}$	$\hat{p}_3' q_3$
Exogenous	$\hat{p}_4' Q_{41}$	$\hat{p}_4' Q_{42}$	$\hat{p}_4' Q_{43}$	$\hat{p}_4' Q_{44}$	$\hat{p}_4' q_4$
Totals	$\mathbf{p}_1' \hat{q}_1$	$\mathbf{p}_2' \hat{q}_2$	$\mathbf{p}_3' \hat{q}_3$	$\mathbf{p}_4' \hat{q}_4$	

$$a_{ij} = \frac{Q_{ij}}{q_j} \Rightarrow Q_{ij} = a_{ij} q_j$$

	1	2	3	4	5
Production	$\hat{p}_1' \mathbf{A}_{11} \hat{q}_1$	0	$\hat{p}_1' \mathbf{A}_{13} \hat{q}_3$	$\hat{p}_1' \mathbf{A}_{14} \hat{q}_4$	$\hat{p}_1' q_1$
Factors	$\hat{p}_2' \mathbf{A}_{21} \hat{q}_1$	0	0	$\hat{p}_2' \mathbf{A}_{24} \hat{q}_4$	$\hat{p}_2' q_2$
Households	0	$\hat{p}_3' \mathbf{A}_{32} \hat{q}_2$	$\hat{p}_3' \mathbf{A}_{33} \hat{q}_3$	$\hat{p}_3' \mathbf{A}_{34} \hat{q}_4$	$\hat{p}_3' q_3$
Exogenous	$\hat{p}_4' \mathbf{A}_{41} \hat{q}_1$	$\hat{p}_4' \mathbf{A}_{42} \hat{q}_2$	$\hat{p}_4' \mathbf{A}_{43} \hat{q}_3$	$\hat{p}_4' \mathbf{A}_{44} \hat{q}_4$	$\hat{p}_4' q_4$
Totals	$\mathbf{p}_1' \hat{q}_1$	$\mathbf{p}_2' \hat{q}_2$	$\mathbf{p}_3' \hat{q}_3$	$\mathbf{p}_4' \hat{q}_4$	



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Price Multiplier

$$\mathbf{p}_1 = \mathbf{p}'_1 \mathbf{A}_{11} + \mathbf{p}'_2 \mathbf{A}_{21} + \mathbf{p}'_4 \mathbf{A}_{41}$$

$$\mathbf{p}_2 = \mathbf{p}'_3 \mathbf{A}_{32} + \mathbf{p}'_4 \mathbf{A}_{42}$$

$$\mathbf{p}_3 = \mathbf{p}'_1 \mathbf{A}_{13} + \mathbf{p}'_3 \mathbf{A}_{33} + \mathbf{p}'_4 \mathbf{A}_{43}$$

$$\mathbf{p}_4 = \mathbf{p}'_1 \mathbf{A}_{14} + \mathbf{p}'_2 \mathbf{A}_{24} + \mathbf{p}'_3 \mathbf{A}_{34} + \mathbf{p}'_4 \mathbf{A}_{44}$$

$$\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$$

$$\mathbf{p}' = \mathbf{p}' \mathbf{A} + \mathbf{v}'$$

$$= \mathbf{v}' [\mathbf{I} - \mathbf{A}_n]^{-1}$$

$$= \mathbf{v}' \mathbf{M}_p = \mathbf{M}_p' \mathbf{v}$$

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13

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Mixed Multiplier

	Endogenous Accounts				Production 3	Exogenous Accounts	Totals	
	Factors	Households	Production 1	Production 2				
Factors	$\mathbf{A}_{nc} = \mathbf{N}_{nc} \cdot \hat{\mathbf{y}}_{nc}^{-1}$				$\mathbf{Q} = \mathbf{N}_Q \cdot \hat{\mathbf{y}}_c^{-1}$		x_{nc}	y_{nc}
Households								
Production 1								
Production 2								
Production 3	$\mathbf{R} = \mathbf{N}_R \cdot \hat{\mathbf{y}}_{nc}^{-1}$				$\mathbf{A}_c = \mathbf{N}_c \cdot \hat{\mathbf{y}}_c^{-1}$		x_c	y_c
Exogenous Accounts	$\mathbf{a}'_{l,nc} = \mathbf{l}'_{nc} \cdot \hat{\mathbf{y}}_{nc}^{-1}$				$\mathbf{a}'_{l,c} = \mathbf{l}'_c \cdot \hat{\mathbf{y}}_c^{-1}$		t	y_x
Totals	\mathbf{y}'_{nc}				\mathbf{y}'_c		y_x	

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14

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The End

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15

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