A Standard Applied General Equilibrium Model: Technical Documentation

STAGE_RDYN Version 2: January 2017[1]

DRAFT

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[1] This model is subject to ongoing developments; this version of the technical document contains details of developments up to the given date. Earlier versions of this model were named differently; the PROVIDE version is the latest of the earlier versions for which documentation is readily available (PROVIDE, 2005).
Various collaborators have contributed to the development of this model. See Appendix 1 on the model's genealogy for details.

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Introduction

This document introduces STAGE_RDYN, which is a general method for the implementation of recursive dynamics in GAMS based CGE models. The implementation discussed in this document is general and relates to the implementation of STAGE_RDYN in the STAGE family of models.

As with all models in the STAGE family the emphasis is on applications relating to developing countries, although this does not preclude using the STAGE models in studies of developed and middle-income countries. The rest of this document is organised as follows. The next section lays out some general principles underpinning STAGE_RDYN. This is followed by some comments on sustainable development and the Millennium Development Goals (MDG). A conceptual approach to recursive dynamics is then presented, followed by a (technical) description of the mechanisms driving the dynamics implemented in STAGE_RDYN; this section includes details of the data requirements.

Recursive Dynamics and STAGE_RDYN

STAGE_DYN is a series of files and conventions developed to implement recursive dynamic applications of all STAGE family models. They exploit the fact that recursive dynamic CGE models are backward-looking, i.e., decisions relating to period *t*+1 are based on the outcomes in period *t*, and hence, in a modelling environment that uses GAMS, recursive dynamic models only require an additional loop, over time, in an otherwise standard experiment file, AND protocols that update various model parameters between time periods. Hence the development of the baseline is a Business as Usual (BaU) scenario, i.e., an experiment.

There are eight major decisions underlying the conventions in the STAGE_DYN.

1. Calibration of the BaU scenario (aka baseline): the default settings involve running a series of experiments for which exogenously derived time-period specific values for model 'variables' are fixed and counterpart model 'parameters' are made flexible. The estimated values for the 'parameters' are then fixed for the simulations, so that without any shocks the previous exogenously fixed 'variables' are replicated. The (crude) alternative of calibrating the BaU scenario using exogenously set of (fixed) model 'parameters' can be used. Both approaches require the user to evaluate carefully the BaU scenario BEFORE running simulations.
2. Time-periods: the model is solved for annual time periods, although the results file can be configured to report results for a subset of time-periods. The limits on computational power that encouraged time periods of multiple years have long gone; the use of annual time-periods allows the user greater flexibility when using the recursive dynamic method. It is also important for the updating of the model's behavioural parameters (see below)
3. Investment and updating capital stocks: the formulation of the STAGE model includes the option of having multiple types of capital, e.g., buildings, plant, vehicles, etc., and therefore the model allows for some capital types to be fixed and others mobile. The default setting for 'fixed' capital, i.e., buildings and activity specific plant, e.g., gas fired electricity generators, is the presumption of a putty-clay model, i.e., once capital stocks are committed to an activity they can only exit that activity by depreciation and/or premature retirement, and the assumption that capital use by each activity is fixed within each time-period. New capital stocks are allocated to each activity for period *t*+1 in shares that reflect the rates of return to capital

in period *t* and the capital stock shares in period *t*, i.e., the allocation principle is one of partial adjustment. For 'flexible' capital items, i.e., vehicles and capital items that at not activity specific, the method of allocating new capital stocks is the same, but a putty-putty model is assumed, i.e., capital items can be reallocated within each time-period. The investment and capital updating codes allow for both fixed and mobile capital.

4. Demographics: the birth and death rates for each RHG are set exogenously based on available data for the current period with an internally generated trend for their evolution over time: the trend adjustment represents the underlying (BaU) changes in the event of no changes in policies. Policy interventions, e.g., changes in per capital education and health expenditures, can cause the birth and death rates to change (increase or decrease) according to exogenously imposed response parameters. The model tracks the demographic profile, i.e., the number of persons in each annual age band, for each RHG: births enter the first year of the profiles with all existing persons aging by one year and deaths reduce the number of persons in the oldest age band. Thus, the demographics modules track the evolution of the composition of the population in natural units.
5. Labour supplies: the demographic modules provide data on the numbers of working age population, and their growths, by each RHG in natural units. But the 'quality' of the labour supplied by each RHG can change. The underlying rates of growth of human capital for each RHG and labour type are set exogenously: these trend adjustments represent the underlying (BaU) changes in the event of no changes in policies. But, changes in education and health expenditures per capita can endogenously increase or decrease the rate of growth of human capital.
6. Calibrating the model's behavioural functions: between each solution the elasticity, shift and share parameters for all non-linear functions, and any parameters calibrated from current period data, are updated. This process of recalibration[2] is a rarely used but important consideration.
7. Macroeconomic closure conditions: the range of macroeconomic closure conditions available within the STAGE_DEV model carry over to the recursive dynamic mode, BUT the implementation of the BaU scenario requires the user to make decisions about macroeconomic closure that

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2 The solution method used by GEMPACK involves an updating of the shares; therefore, the GEMPACK equivalent of recalibration is 'automatic".

influence the range of options available to the user when running recursive dynamic simulations.

8. Market clearing conditions: the range market clearing available within the STAGE_DEV model carry over to the recursive dynamic mode, BUT the choices when implementing the BaU scenario influence the range of options available to the user when running recursive dynamic simulations.

All recursive dynamic CGE model simulations are constrained by the absence of inter temporal optimising behaviours. It is therefore critical for users to understand the serious limitations of recursive dynamic CGE models despite their appeal to policy decision-makers.

It is important to appreciate the correct interpretation of the BaU scenario and simulation scenarios in CGE models. In reference to the results from recursive dynamic CGE models it is not uncommon to hear them referred to as ‘forecasts’; the use of this term is often associated with results from the MONASH and USAGE models and the work of Peter Dixon (see Dixon and Rimmer, 2002 and 2009) but is used very commonly, especially in the context of consultancy. The argument advanced here, and consequently the approach adopted by the STAGE_RDYN model, is that CGE models cannot produce meaningful whole economy economic forecasts and hence should not be represented as economic forecasting models, where an economic forecast is defined as an estimate or projection of an economic system at some time in the future.[3] Therefore they cannot tell decision-makers what the state of an economy will be at some time in the future.

Underlying this argument are four points. First, whole economy CGE models are Walrasian and hence exclude macroeconomic features, e.g., assets and financial markets are absent. Second, recursive dynamic CGE models explicitly adopt the principles underlying models of economic growth and implicitly assume that the economies are in some form of steady-state and follows some form of optimal growth path. Third, all changes in GDP, or some other macroeconomic aggregate, that are not explained by changes in inputs are assume to result from some unexplained form of productivity growth; aka a ‘measure of our ignorance’ or the ‘residual’. And fourth, real world features of economies, such as uncertainty, risk and the ‘animal spirits of entrepreneurs’, are unknowns that cannot be identified or quantified.

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3 It can be validly argued that economic forecasting with any form of economic model is, at best, a dubious activity.

But, while this class of models are not, and never can be, forecasting models, they can provide insights into the forces that economic shocks and policy changes may generate assuming there are no changes in the macroeconomy, e.g., asset and finance markets. Critical to an interpretation and understanding of the results from recursive dynamic CGE simulations are the results from the BaU scenario. Analysts need to pay attention to these results before they can understand the deviations from the BaU scenario generated by simulations.

Conceptualising an Approach

These arguments suggest, strongly, that the inclusion of SD targets within CGE models should be based on the arguments in the economic literature on development: this means within the context of the basic needs, human development and poverty/entitlements literatures. Three strands stand out. In each strand, it is possible to conceptualise the approach in terms of functions defining reductions in resources, e.g., the depreciation of physical capital, labour 'retiring' from the workforce, etc., and those defining increases in resources, e.g., the gross accumulation of physical and human capital, gross improvements in health, etc. Combining these together to generate net (positive or negative) changes in available resources.

In stylised form this approach can be presented as a form of growth accounting relationship. Using a macroeconomic form and two argument (input) Cobb-Douglas functional form for convenience[4]

$$Y = \alpha_y \left(\alpha_k K\right)^{\beta_k} \left(\alpha_l L\right)^{\beta_l} \quad (1)$$

which can be expressed in terms of the determinants of total factor productivity (TFP) as

$$TFP = \frac{d\alpha_y / dt}{\alpha_y} = \frac{dY / dt}{Y} - \left(\omega_k . \frac{dK/dt}{K} + \omega_k . \frac{d\alpha_k/dt}{\alpha_k}\right) - \left(\omega_l . \frac{dL/dt}{L} + \omega_l . \frac{d\alpha_l/dt}{\alpha_l}\right) \quad (2)$$

where Y is output, K and L are the stocks of capital and labour (respectively) and α_y, α_k and α_l are the efficiency parameters for output, capital and labour (respectively). Equation 2 can be interpreted as defining total factor productivity growth (the unknown) as the difference between the rate of growth of output less the weighted growth of the inputs, where input growth for both inputs is decomposed into terms relating to the growth of the stocks and the flow of services from each unit of the stocks. This highlights the need to always conceive of inputs in terms of both the stock of any input and the rate of flow of services from each unit of the stocks.

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4 This follows from Solow (1957) and can easily be generalised using any twice differentiable (well behaved) production function.

Human resources

This derives from the argument that most persons who are poor are, overwhelmingly, dependent upon the sale of labour services to generate incomes; therefore, the primary focus should be upon those attributes that determine the value of their labour. Simplistically, this can be reduced to the skills individuals possess and their health; the former determining the attributes their labour services provide while the latter determines how long they might work (influenced strongly by mortality) and the intensity with which they can work (influenced strongly by morbidity). Also relevant are the characteristics of populations; the growth rate (fertility), the death rates and the age profile (dependency ratios).

Ultimately, there are arguments for the development of demographic profiles that encompass information about the ages, genders, skills, etc., of populations that can then be used to generate demographic transition matrices (see Stone, 1984) that provide detailed data about the human resources available to countries.

A major argument against this approach is that it defines human resources in terms of the productive capacities of labour services, i.e., as an input. This runs counter to the standard economic argument that people render labour services because of the incomes so generated and the quality of life those incomes provide, i.e., rendering labour services is a means to an end rather than an end.

Skills

Classifying labour types by the inherent skills rather than by occupations is likely to provide a better method for identifying the extent to which labour can transition between activities. But defining inherent skills is problematic since for any individual there are unlikely to be deterministic relationships between the productivity of that person and simple quantifiable measures of skills. However, when dealing with populations or relatively large groups of persons there is a reasonable case to be made for using standard proxy measures of skills; namely measures of educational attainment.

<u>Education</u>

Measures of educational attainment have been used extensively in the empirical literature of growth accounting and endogenous growth (see Barro and Lee, 2012). Typically, the available data are expressed in terms of the number of years of schooling and/or the stage of schooling reached, e.g., primary, secondary, tertiary, etc. Ideally it could be argued that the measures should be expressed in units that measure attainment, e.g., literacy and numeracy levels, school certificate and degree grades, etc.; but this raises issues about cross regional and generational comparability. Ultimately the choice is likely to be driven by the availability of data as much as by some ideal standard.[5]

<u>Training</u>

It is more difficult to measure the level of training in task specific skills. While educational attainment may represent a measure of the potential skills of an individual it may not be a good measure of the actual skills and knowledge. It is likely that the actual skills and knowledge will increase with experience and 'learning by doing' (see Arrow, 1964); note however, that these relationships may not be a simple function of age since evolving labour market conditions are likely to see persons changing employment through their lives, e.g., as a consequence of rural-urban migration.

It is difficult to identify robust measures of 'actual skills and knowledge'. It may be that in addition to using educational attainment to define labour categories it may be appropriate to use age bands to give some, albeit crude proxy.

<u>Health6</u>

The ability of persons to provide labour services are inevitably influenced by their health status. When including health indicators, it is helpful to distinguish between mortality and morbidity: mortality can be thought of as a 0:1 condition – not alive or alive – whereas morbidity is a continuous measure given that a person is alive.

5 There are links between the interpretation of educational attainment measures and health. For instance, children suffering chronic malnutrition at critical phases of their development can have impaired future mental capacities (see Dasgupta, 1993).

6 Dasgupta (1993) provides a good overview of some of the key issues relating to health.

Malaria presents a classic illustration of the importance of the distinction between mortality and morbidity and the complexity of the issues it raises. Malaria in childhood can result in death; but those who survive acquire a degree of immunity that means that later bouts of malaria typically increase absences from work, albeit with a shortening of life expectancy. Thus, morbidity effects are most important, from the labour market perspective, for those who survive childhood malaria. But persons who first acquire malaria as adults' experience very high death rates, i.e., the mortality effects dominate, which has knock-on implications in terms of dependency rates and prospects for their offspring.

Mortality

In this sense the mortality rate can be conceived of as a direct determinant of the size of the population, i.e., the rate at which the **stock** of the population depreciates. If mortality rates are linked simply to the size of the labour force, then they are just measures of the rates at which persons exit the labour force. But, because persons exit the labour force does not mean they die; rather they may be elderly persons' dependent on other family members for their survival; this raises issues as to the extent to which reductions in mortality rates may generate increases in (elderly) dependency ratios.[7]

Ultimately mortality rates can be thought of as one of the arguments determining the quantities/stocks of labour available. Thus, improvements in general health status can be thought of as, *ceteris paribus*, increasing the population/labour force.

Morbidity

Morbidity is more complex, if only because its measures must be continuous rather than discrete variables. Ill health can reduce an individual's ability to carry out manual labour and/or increase the number and extent of absences from employment.

Ultimately morbidity rates can be thought of as one of the arguments determining the quality of labour services available. Thus, improvements in general health status can be thought of, *ceteris paribus*, as enhancing the quality of labour.

7 In the context of extended families in developing countries the issues are complicated by the extent to which elderly persons can release other family members for directly productive labour activities by taking on domestic and child care activities.

<u>Population</u>

Population size is clearly an important consideration; it provides a cap on the availability of labour stocks. But the relationship between population and the size of the labour force can be complex. In a 'steady state', the birth and mortality rates and the life expectancy at birth will be constant and hence, *ceteris paribus*, there will, in the long run, be a constant ratio between the sizes of the population and labour force. But such a 'steady state' is not consistent with expectations of development where changes in birth and mortality rates and the life expectancy at birth may be regarded as key indicators of development.

A problem many developing countries face is that the classic demographic transition, from high birth and death rates to low birth and death rates via a period of high birth and falling death rates and the falling birth rates and low death rates, driven by domestic/endogenous economic growth has changed. Many developing countries now face prolonged periods of high birth rates and lower death rates due to exogenous factors (see Deaton, 2013), and hence rapid population growth.[8]

Fertility

Typically, fertility rates are assumed to be exogenous in economic models; reflecting a presumption that economists do not have good behavioural explanations for fertility rates. One approach to endogenising fertility rates is to assume that they are inversely related to the level and/or growth of income per capita. But fertility rates alone are not adequate since they need to be related to the proportions of the population in child bearing age groups.

Dependency ratios

In one sense dependency ratios are the inverse of labour force participation rates; thus, tracking dependency ratios and labour force participation rates are essentially the same activities. Ultimately this reduces to the tracking of the demographic profiles for households. In its simplest form a demographic profile could be broken down into five categories

1. Dependent children
2. (Compulsory) School age children

[8] One consequence of this is the large proportions of populations in childbearing age groups that will in subsequent years results in a large proportion of dependent elderly persons, e.g., in much of the middle east. In China, following the one-child policy, a large proportion of dependent elderly persons is already becoming an issue (as it has already done in much of the developed world).

  - Primary school
  - Secondary school
3. Young adults
   - In work
   - In education and/or training
4. Working age adults
   - In work
   - Not in work
   - Not in the labour force
5. Dependent adults.

Tracking the evolution of a demographic profile using changing birth and mortality rates and the life expectancies at birth provides a method for defining the relationship between population and the labour force.

Manufactured resources

Most of the literature relating to manufactured resources emphasises physical capital and its accumulation. This however neglects two important considerations; first, the simple/crude accumulation of physical capital amounts to the 'piling of wooden ploughs on top of wooden ploughs' because it neglects the matter of the changing quality of physical capital, and second, changes in the quality of intermediate inputs and their impact of productivity.

Physical capital

The standard approach here is simply to record gross investment and economic depreciation to derive estimates of net investment. This is a well-developed method but often begs the question about the determination of the rates of gross investment – savings 'driven' or 'driven' by the expectations of entrepreneurs – and the problems of measurement.

Embodied technologies

The concept of embodied technologies is intrinsically simple; so-called Solow neutral technical progress.

Physical Capital

New machines are argued to contain the new technologies and hence to involve different input mixes reflecting the technologies embodied in the capital goods. One approach to this issue is to record the 'vintages' of investment goods and to assume that the productivities of capital goods are monotonically related to their ages; this amounts to running some form of perpetual inventory that can be translated into an aggregate used in the model.[9]

Intermediate Inputs

This issue appears to be largely unaddressed and no clear statement of approaches in CGE models can be found, which perhaps reflects the fact that the modeling of intermediate input demand in CGE models remains very simple. At an aggregate level, certain CGE models allow for the inclusion of aggregate activity level productivity factors for intermediate inputs, but the method is largely *ad hoc*.

9 This does impose restrictions of the operation of substitution across activities.

Natural and environmental resources

Natural resources and Environmental issues are increasing topics of interest in national accounts and CGE models. In most of the literature the emphasis is on the development of inventories that record extraction and replenishment rates.

Finite (mineral) resources

Typically, mineral resources are available in fixed quantities determined by nature and evolution, although the distinction between known reserves and actual reserves needs to be noted. The standard approach is to record the extraction rate in each year for each natural resource, which given the known reserves in the base year defines the amounts available for future use. However, problems arise with respect to the valuation of remaining reserves[10] and their incorporation into decision making.

Renewable resources

Renewable resources are more readily accommodated into recursive dynamic models since they can be conceived of in terms of the maximum amounts of the renewable resource available in any period. Water is an obvious example: Luckmann and McDonald (2014) endogenise water as a produced factor of production in a CGE model where natural water resources, including seawater, require other inputs to render it usable and where waste water can be recycled.

Air quality and Climate

Within single country CGE models air quality and climate effects are overwhelmingly exogenously determined. Thus, they typically enter the model as simple exogenous variables.

[10] Among other issues this raises questions over the appropriate definition of discount rates and the extent to which discount rates 'discriminate' against future generations.

Implementation of the Recursive Dynamics

In any backward-looking model agents respond to incentives in the current period to determine their actions in the next period. Thus, if the price of some factor increases, agents should try to increase the quantity of that factor available to them; but if agents are assumed to respond fully to such changes in incentives the resultant system could experience wild fluctuations, e.g., the 'hog cycle'. The favoured approach is that of partial adjustment, where the extent of adjustment is determined by exogenous adjustment parameters (*mu_**); consequently, if a single shock is introduced the system will adjust to this shock overtime and that several time periods are required before the system returns to a long run equilibrium state.[11] The time taken to return the long run equilibrium will depend on adjustment rates (parameters) and the size and nature of the shock.

The standard presumption is that the model is solved for annual steps under the maintained assumption that agents make decisions about resource allocations at the start of the **current** period, e.g., the amount of physical capital available to each activity. These decisions are then revised based on the outcomes, primarily prices, in the **current** period to determine the decisions about resource allocation at the start of the **next** period.[12]

It is relevant to note that the existence of stock changes in base data sets implies that the base data do not represent a long run equilibrium situation. Therefore, it can be appropriate to include a trend rate that eliminates the stock changes over time. However, while this option achieves an objective – getting rid of stock changes – it does not, and cannot, address the fundamental problem about stock changes: namely that their existence demonstrates that the initial database may not have been in a steady-state equilibrium.

<u>Demographics</u>

At its simplest level demographics is about little more than changes in the size of populations. On a slightly more complex level it is concerned with how the structures of populations change with respect to age and gender profiles, while at a still more complex level it involves

11 A solution from a comparative static model after a shock represents such a long run equilibrium.

12 This approach is rooted in the tradition of Harrod-Domar growth models where backward-looking decisions are important. This does raise important question about the appropriate definitions for investment functions (see Khan, ??).

tracking how populations evolve in terms of their ownership of assets through *inter alia* the processes of education, training and health. The approach embedded in this version of STAGE_RDYN is a compromise that seeks to limit the data requirements for its implementation while providing enough additional information to capture aspects of the evolution of the populations of and labour supplies by RHGs.[13]

The demographics are implemented through two include files in the recursive dynamic codes[14]:

1. '*stg2_dyn_demog_14.inc*': this file defines all the parameter and other symbols referring to demography used in STAGE_RDYN and assigns their initial values, there is an option to load additional demographic specific data within this file.

2. '*stg2_dyn_pop_14.inc*': this file is implemented within the time loop when running the recursive dynamic model. It implements any trend rates of change in birth and death rates by RHG, and then further adjusts the birth and death rates by RHG in response to changes in economic incentives – the volumes of healthcare and education expenditures by the government. The age profile of the population by RHG is updated given the birth and death rates.

The derivation of the population profiles depends upon the information available when the databases are compiled. If the RHGs are defined using data derived from household income and expenditure surveys (HIES), or censuses that can be linked to the HIES, then there is usually enough data to estimate the numbers of persons in certain age bands and from those data to infer the numbers in each annual cohort. Given the numbers in each annual cohort it is also possible to derive estimates of the numbers of children borne each year and hence the estimates of the crude birth rate adjusted for early year infant mortality.

The methods coded assume very limited information. The numbers in the age group - 0-15, 16-55, and 55+ - the crude birth and death rates; these data are freely available from the World Bank's database.

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13 It is pertinent to note that while demographics were not an explicit component of the contract they have been added because otherwise he endogenous functional distribution of income element of the STAGE_DEV model in the contract would have been substantially weakened.

14 GAMS requires that symbols are declared and defined outside off conditional statements, e.g., loops. Because the changes through time are implemented inside a (time) loop it is necessary to have two files – one outside and one inside the loop.

The process by which birth and death rates are adjusted uses a constant elasticity function to generate adjustment factors, e.g.,

$$factor_h^t = factor_h^{t-1} * \left\{ \frac{q_{h,i}^t}{q_{h,i}^{t-1}} \right\}^{\mu_h^t} \tag{4.3}$$

where $factor_h^t$ is the adjustment factor for RHG h in period *t*, $q_{h,i}^t$ is the volume the intervention per head of population in the RHG and μ_h^t is the responsiveness of the rate to the change in the volume of per capital intervention. Then the birth or death rates are updated, when there are no trend changes in the rate, using

$$rate_h^t = \left(factor_h^t \right) * rate_h^{t+1} \qquad \textbf{OR} \qquad rate_h^t = \left(\frac{1}{factor_h^t} \right) * rate_h^{t+1}. \tag{4.4}$$

The different formulae in (4.4) depend upon whether it is assumed the intervention increases or reduces the rate. Thus, for instance, increases in healthcare expenditure may be assumed to increase the **effective** birth rates because of reduced foetal and early years (under 5) deaths, while increases in education expenditure may be assumed to decrease the **effective** birth rates because of increases in the opportunity costs of pregnancy and female empowerment.

Note how Eqn 4.3 ensures that the impact of changes in the volume of intervention per capita is cumulative. If the volume of intervention per capita increases continuously so will the adjustment factor, because the ratio of the current to the previous volume of intervention is greater than one, but if at any time the volume of intervention declines for one period to the next the ratio of the current to the previous volume of intervention will be less than one and the adjustment factor will reduce.

Having determined the birth and death rates for period *t*, the populations by each RHG are updated. Given the populations of each RHG in (*t*-1) the number of deaths and births can be derived. Deaths are recorded as removing the enough persons from the oldest annual cohorts of the populations, then the remaining populations in (*t*-1) are all aged by one annual cohort and births are recorded as populating the youngest (zero to one) cohort. Thus, the profiles of the populations for each RHG evolve, and with them the cohort sizes for dependent children, children of school age, dependent (old) adults and working aged adults.

Human Capital Accumulation

Human capital accumulation raises several conceptual issues and problems with the classification of working population. It has become common, for economists, to equate human capital accumulation with educational attainment, e.g., Barro (2001) and Barro and Lee (1993 and 2012), but there is also a longstanding current in the human capital literature about the role of 'learning-by-doing' (Arrow, 1962) that highlights the role of human capital accumulation through experience, practice and on-going learning. Thus, while educational attainment may, on average, provide indications of the potential productivity of 'new' members of the workforce, it may be a poor indicator of the average flow of labour services from any given stock of labour. One approach may be to suggest that groups of individuals enter the workforce with a specific flow of labour services associated with their terminal educational attainment, and thereafter they continue to acquire skills that increase their productivity until age starts to degrade their productivity.

But the empirical basis available to calibrate such evolving labour productivity is sparse, to say the least. Moreover, there is a classification problem. Very often, following ILO convention, workers are classified by their occupations not their levels of skills; so even if data can be collected on the educational attainment of labour services owned by households, it is rare to find data on the employment of labour by educational attainment by activities.

The approach taken for STAGE_RDYN is, again, a pragmatic one that seeks a compromise between data requirements and behavioural precision. The process can be conceived of as consisting of three elements; a trend rate (based on historical evidence) of human capital accumulation, educational expenditure that raises skill levels and health expenditures that enhance the ability of workers to generate labour services. All three elements contribute to increases in the efficiency of workers so that the *de facto* flow of services from any given stock of workers increase. As such, the model records two measures of labour; the physical numbers of workers of each type supplied by each RHG and the efficiency factor for each type of labour supplied by each RHG. The former defines the ownership of each type of labour by each RHG in natural units – person hours – while the latter defines the flows of each type of labour service available for production.

The education component is defined as

$$eff\_l\_ed_{h,l}^{t} = eff\_l\_ed_{h,l}^{t-1} * \left(\frac{educ_{h}^{t}}{educ_{h}^{t-1}}\right)^{\mu_{h,t}^{ed\_l}} \quad (54.5)$$

where $eff\_l\_ed_{h,l}^{t}$ is the efficiency factor due to education for labour of type *l* provided by RHG *h* in period *t*; $educ_{h}^{t}$ is volume of education services per child in RHG *h* in period t and $\mu_{h,t}^{ed\_l}$ is the responsiveness of the efficiency factor to changes in the per child volume of education services. Similarly, the health component is defined as

$$eff\_l\_he_{h,l}^{t} = eff\_l\_he_{h,l}^{t-1} * \left(\frac{heal_{h}^{t}}{heal_{h}^{t-1}}\right)^{\mu_{h,t}^{he\_l}} \quad (4.6)$$

where $eff\_l\_he_{h,l}^{t}$ is the efficiency factor for labour due to health of type *l* provided by RHG *h* in period *t*; $heal_{h}^{t}$ is volume of health services per person in RHG *h* in period t and $\mu_{h,t}^{he\_l}$ is the responsiveness of the efficiency factor to changes in the per person volume of education services. Note how, in both cases, the efficiency factor is cumulative as in the case of the demographics.

The enhancement to productivity due to education and healthcare expenditures are assumed to be multiplication, hence

$$fsia_{h,l}^{t} = fsia_{h,l}^{t-1} * eff\_l\_ed_{h,l}^{t} * eff\_l\_he_{h,l}^{t} \quad (4.7)$$

where $fsia_{h,l}^{t}$ is the supply of labour services of type *l* by RHG *h* in period *t*.

The trend rate of human capital accumulation can be conceived of as ‘learning-by-doing’ and as being cumulative, thereby further enhancing the efficiency factor.

<u>Physical Capital Accumulation</u>

The updating of physical capital follows a standard perpetual inventory model. At the end of each period the existing capital is depreciated using economic depreciation rates that are functions of the intensity of use of the stock in the previous period[15] and then gross investment is added to determine the stock of capital in the next period. It is assumed that new

[15] This done by basing depreciation on the value of returns to capital in the previous period rather than the quantity of capital, i.e., the rate of return provides a proxy for the intensity of use.

investment is delivered to activities in the same ratio as the existing capital stock **unless** the relative rates of return to capital by activity have changed. If the relative rates of return have changed, the pattern of investment by activity is different to the existing capital stock but does not wholly adjust to the new rates of return, i.e., there is a partial adjustment process. It is typically assumed that the stock:flow ratio is constant, i.e., the quality of the capital is constant.

Capital can be divided into ‘fixed’ and ‘flexible’ capital. It is assumed that once ‘fixed’ capital is located within an activity it can only exit that activity by economic and/or accelerated depreciation; examples of such capital would include power generating plants, machines for specific operations etc. This reflects the fact that the adjustment processes with respect to ‘fixed’ capital can be slow.[16] On the other hand, ‘flexible’ capital enters a pool that can be allocated across activities in the current period in response to changes in the rates of return.

With multiple forms of capital goods, the commodities required to produce each form of capital will differ. But the commodities used for investment in capital for the next period must be produced in the current period, which mean that the demand for commodities for investment in period (*t*+1) must be decided at the end of period (*t*-1) so that they can be produced in period *t*.

The first step is to convert investment expenditures into a volume of new capital. It important to be careful to ensure that there is no confusion between the value of capital stocks and the rate of return; this is important since the units in which the value of capital stocks and the rates of return are measured need to be consistent. The volume of new capital is defined as

$$gfcf_k^t = invest\_f_k^t / pk_k^t \tag{4.8}$$

where $gfcf_k^t$ is the gross fixed capital stock of type *k* in period *t*, $invest\_f_k^t$ is the value of investment in *k* in period *t*, which is defined as the summation of the expenditures on the different commodities (*QINVD*) used to produce capital of type *k*, and pk_k^t is the price index of capital of type *k* produced in period *t*.

16 This is one of the reasons to prefer solving the model year by year rather than in, say, 5 or 10 year periods.

Then the new capital is allocated across activities. The investment shares for capital of type *k* by activity *a* are

$$inv\_sh\_ka_{k,a}^{t} = cap\_sh\_ka_{k,a}^{t-1} * \left(1 + \mu_a^t * \left(wfa\_rel_{k,a}^{t-1} - 1\right)\right) \tag{4.9}$$

where $inv\_sh\_ka_{k,a}^{t}$ is the share of the volume of investment of capital type *k* by activity type *a* in period *t*, $cap\_sh\_ka_{k,a}^{t-1}$ is the share of the volume of capital of type *k* used by activity of type *a* in period (*t*-1), $wfa\_rel_{k,a}^{t-1}$ is the relative rates of return to capital of type *k* in activity of type *a* in period (*t*-1) and μ_a^t is the partial adjustment (response) parameter for activity of type *a*. The quantities of new capital allocated to each activity $\left(d\_kap\_ka_{k,a}^{t}\right)$ are

$$d\_kap\_ka_{k,a}^{t} = inv\_sh\_ka_{k,a}^{t} * gfcf_k^t \tag{4.10}$$

and the new quantities of capital by activity for period are

$$FD_{k,a}^{t} = \left(FD_{k,a}^{t} * \left(1 - deprec_k\right)\right) + d\_kap\_ka_{k,a}^{t} \tag{4.11}$$

where $FD_{k,a}^{t}$ is the stock of capital of type *k* available to activity *a* at the start of period t and $deprec_k$ is the depreciation rate for capital of type *k*.

Note that the Eqn 4.6 only defines the availability of capital of type *k* to activity *a* at the start of period *t*. If capital of *k* is made activity specific (fixed) then the quantity of capital of type *k* in activity *a* at the end of *t* will be the same, but if capital of type *k* is deemed flexible then capital of type *k* will be reallocated within period *t*. Thus, this allocation method works for either fixed or flexible capital.[17],[18]

However, it is not enough to allocate investment capital to each activity, it is also necessary to define the ownership of new capital by RHG so that the functional distribution of the incomes from capital goods is consistent with savings by domestic institutions. Given the savings rates for institutions, which can be either endogenous or exogenous variables, and the

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17 If capital of type *k* is mobile, then is the relative rates of return to capital of type *k* in activity of type *a* in periods *t* and (*t*-1) will be identical and so will be the investment and capital shares in (4.3).

18 The alternative approach of defining the total supply of capital of type *k* and then allowing the solution to define its allocation in use produces an identical allocation. The difficulty with implementing this through the total supply is that it limits the factor market clearing options available when using the model.

incomes and income tax rates from the period specific solutions, the value of savings by each institution can be calculated. The ownership of capital by each institution in the next period is

$$FSI_{ins,k}^{t} = \left(FSI_{ins,k}^{t-1} * \left(1 - deprec_k\right)\right) + d\_kap\_ins_{ins,k}^{t} \qquad (4.12)$$

where $FSI_{ins,k}^{t}$ is the supply of capital of type *k* by institution (*ins*) and $d\_kap\_ins_{ins,k}^{t}$ is the change in the supply of capital by institution. Note how the depreciation rates are specific to the type of capital and the same as those used when determining the stocks of capital available to activities.

Finally, it is necessary to identify the pattern of capital goods produced in the next period; this is a necessary bridge between the identification of savings at the level of the institution and the composition of capital stocks. The key assumption is that expectations about the pattern of capital good production in the current period for investment in the next period partially adjust to the rates of return to the types of capital, i.e., the pattern of capital goods production for the next period is determined at the beginning of the current period. Thus

$$inv\_sh\_k_k^t = cap\_sh\_k_k^{t-1} * \left(1 + \mu\_k_k^t * \left(wf\_rel_k^{t-1} - 1\right)\right) \qquad (4.13)$$

where $inv\_sh\_k_k^t$ is the share of investment devoted to capital of type *k*, $cap\_sh\_k_k^{t-1}$ is the share of capital of type *k* in the previous period, $wf\_rel_k^{t-1}$ is the relative rate of return to capital of type *k* and $\mu\_k_k^t$ is the partial adjustment (response) parameter for capital of type *k*. Thereafter the base levels of investment volume for the next period can be defined as

$$qinvb_i^t = \sum_{i,k\$map\_i\_k} inv\_sh\_k_k * \sum_i qinvb_i^{t-1} \qquad (4.14)$$

where $qinvb_i^t$ is the share of investment funds devoted to investment account *i*. Note how there is a unique mapping between investment accounts and capital goods defined by the set $map\_i\_k_{i,k}$.

Business as Usual Scenario (Baseline)

The implementation of the BaU scenario is an experiment in which the shocks are defined as the time paths for a series of exogenous variables with appropriate settings for the macroeconomic closure rules and the factor market clearing conditions. The range of exogenous variables that can be set depends overwhelmingly on the perspective of the analyst and, possibly, the requirements of the policy makers. Typically, these might include forecasts for several macroeconomic aggregates, e.g., GDP, investment, household and government consumption, internal and external balances, etc., plus, possibly, some presumptions about relative rates of activity and/or factor specific productivity growth. There is no unique or correct set of exogenous variables relative rates of productivity growth, so while certain options may be preferred by one analyst, another may well prefer a different set of options.

Having defined the exogenous variables, it is necessary to pair each of these with an endogenous variable; an action that is *de facto* the determination of the appropriate setting of the macroeconomic closure rules. If any relative rates of productivity growth are fixed, then it is necessary to make sure the settings for the derivation of productivity adjustment are set correctly.

The BaU scenario can then be implemented and the results can be saved; note since this is an experiment the process for collecting the results is identical to that for a standard recursive dynamic experiment. It is important that once the BaU scenario has been implemented that the results are analysed **as if** they are a standard experiment, i.e., they need to be checked for logical coherence – the evolution of the economy – and the economic implications. There is no point to running recursive dynamic experiments off a BaU/baseline that is incoherent and/or produces unsustainable economic implications.

Once the BaU scenario has been implemented and checked the model can be configured to run policy simulations. This requires changing the closure rules so that the endogenous variables for the BaU scenario are exogenous variables in the policy experiments and that the exogenous variables for the BaU scenario are endogenous variables in the policy experiments; the requisite data can be accessed form the results or, as in STAGE_RDYN from a specific GDX file that records the required endogenous variables for the BaU scenario. It is important to make sure any relative rates of productivity growth are carried over into the policy experiment settings.

Collection of the Results

The collection of results follows the same principles adopted for the STAGE/GLOBE family of models. All the values for the levels of the variables after each solution are record as parameters indexed on the simulation set, the closure set, the sensitivity analyses set and the time set. Thereafter, derived results are calculated using the levels results and base results, in the recursive dynamic case the base results include not only the value of all variables in the first year and all model parameters, but also the BaU results. The BaU results are important because the standard method for presenting recursive dynamic results is in terms of the deviations from the BaU scenario.

Appendix 1: STAGE Model Genalogy

The STAGE model started life in the mid 1990s. After initial (futile) struggles with the Cameroon CGE model then in the GAMS library Sherman Robinson gave Scott a copy of the single country CGE model developed for the US Department of Agriculture's (USDA) Economic Research Service (ERS) under Sherman's leadership (Robinson *et al*., 1990; Kilkenny, 1991). The USDA model was based on an input-output representation of the inter industry transactions that limited the applicability of the model for the analyses of the decisions made by multi-product activities. This concern was raised with Sherman and Hans Lofgren in the late 1990s[19]; this problem was addressed by Hans and Sherman and a copy of the solution was shared with Scott. The developments by Hans and Sherman at IFPRI ultimately resulted in the production of the IFPRI standard model in 2001 (Lofgren *et al*., 2001). Consequently the IFPRI standard and STAGE models share a common heritage and a number of features although there also differences.

The PROVIDE project model (McDonald, 2003) was developed during a programme of research funded by the South African National and Regional Departments of Agriculture. The development of the PROVIDE model benefited from contributions by Cecilia Punt, Melt van Schoor, Lindsay Chant and Kalie Pauw

The PROVIDE project model development into the STAGE model as part of the process of developing the GLOBE model from 2002 with Karen Thierfelder. The GLOBE model used a simplified variant of the STAGE model as the basis for the development of the within country/region behavioural equations. This process generated some changes in behavioural relationship, code structure, methods for analyzing results and notation. Consequently in 2005 the STAGE 1 model was consolidated from previous models and made open source with some revisions in 2009.

The STAGE 2 model is a consolidation of model developments since 2009. It embodies contributions made by Karen Thierfelder, Emanuele Ferrari and Emerta Aragie.

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19 The issue had become relevant when estimating the implications of BSE (McDonald and Roberts, 1998).

The STAGE model is part of a suite of models that include a global model (GLOBE model) and a range of teaching models – the SMOD suite. All these model use a (overwhelmingly) common set of notation and formats.